About the HELM Project

HELM (Helping Engineers Learn Mathematics) materials were the outcome of a three-year curriculum development project undertaken by a consortium of five English universities led by Loughborough University, funded by the Higher Education Funding Council for England under the Fund for the Development of Teaching and Learning for the period October 2002 September 2005.

HELM aims to enhance the mathematical education of engineering undergraduates through a range of flexible learning resources in the form of Workbooks and web-delivered interactive segments.

HELM supports two CAA regimes: an integrated web-delivered implementation and a CD-based version.

HELM learning resources have been produced primarily by teams of writers at six universities:
Hull, Loughborough, Manchester, Newcastle, Reading, Sunderland.

HELM gratefully acknowledges the valuable support of colleagues at the following universities and colleges involved in the critical reading, trialling, enhancement and revision of the learning materials: Aston, Bournemouth & Poole College, Cambridge, City, Glamorgan, Glasgow, Glasgow Caledonian, Glenrothes Institute of Applied Technology, Harper Adams University College, Hertfordshire, Leicester, Liverpool, London Metropolitan, Moray College, Northumbria, Nottingham, Nottingham Trent, Oxford Brookes, Plymouth, Portsmouth, Queens Belfast, Robert Gordon, Royal Forest of Dean College, Salford, Sligo Institute of Technology, Southampton, Southampton Institute, Surrey, Teesside, Ulster, University of Wales Institute Cardiff, West Kingsway College (London), West Notts College.

HELM Contacts:

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HELM Workbooks List

1	Basic Algebra	26	Functions of a Complex Variable
2	Basic Functions	27	Multiple Integration
3	Equations, Inequalities & Partial Fractions	28	Differential Vector Calculus
4	Trigonometry	29	Integral Vector Calculus
5	Functions and Modelling	30	Introduction to Numerical Methods
6	Exponential and Logarithmic Functions	31	Numerical Methods of Approximation
7	Matrices	32	Numerical Initial Value Problems
8	Matrix Solution of Equations	33	Numerical Boundary Value Problems
9	Vectors	34	Modelling Motion
10	Complex Numbers	35	Sets and Probability
11	Differentiation	36	Descriptive Statistics
12	Applications of Differentiation	37	Discrete Probability Distributions
13	Integration	38	Continuous Probability Distributions
14	Applications of Integration 1	39	The Normal Distribution
15	Applications of Integration 2	40	Sampling Distributions and Estimation
16	Sequences and Series	41	Hypothesis Testing
17	Conics and Polar Coordinates	42	Goodness of Fit and Contingency Tables
18	Functions of Several Variables	43	Regression and Correlation
19	Differential Equations	44	Analysis of Variance
20	Laplace Transforms	45	Non-parametric Statistics
21	z-Transforms	46	Reliability and Quality Control
22	Eigenvalues and Eigenvectors	47	Mathematics and Physics Miscellany
23	Fourier Series	48	Engineering Case Studies
24	Fourier Transforms	49	Student's Guide
25	Partial Differential Equations	50	Tutor's Guide

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Tutor's Guide

Introduction to HELM $\boxed{50.1}$

1. The 'Mathematics Problem' in the engineering context

In recent years the mathematical preparedness of students embarking upon science and engineering degree programmes has been the subject of close scrutiny in numerous research reports [for example 1, 2, 3, 4], and the topic has been debated at many conferences [for example, 5, 6, 7], with some disturbing conclusions. A common theme running through all this work is that these students are, on the whole, significantly mathematically weaker than students coming to university a decade or so ago. Traditionally, students embarking upon engineering degree programmes had to demonstrate a very respectable competence in mathematics and physics through the achievement of good A level grades. However, the recent past has seen a widening of access for understandable financial and social reasons.

Academic staff face the conundrum of needing to devote less time (or at least less energy) to teaching and related administration whilst tailoring courses adequately to the needs of the wide range of individual students in their care.

In teaching mathematics to engineers, the primary vehicle of transmission still remains the lecture. It is particularly efficient with large groups, although interaction is then especially difficult. Syllabus requirements may, however, encourage lecturers to try to cover too much material thus hindering student understanding. In any case, lectures are not the best place for transmitting a great deal of technical information, especially when students are trying to copy it from an OHP or writing board. Understanding becomes more difficult and many errors of transcription are made. The lecture notes of an average student often bear only passing resemblance to what was actually delivered, which can be crucial to understanding in mathematics.

Much material is now available in electronic form, on CD or on the web, and computer aided learning (Interactive Learning Segments) has tremendous potential to assist the learning process. Information can be presented in written format (on screen) which can be supplemented with animation, with commentary and with video. Interaction adds more interest. It can be delivered locally on an intranet network or over the web. It can be recorded on a CD and used by students not wishing (or able) to use an internet connection. However, the development and start-up costs are extremely high, both in time and in resources. There is much anecdotal evidence to suggest that even though there is quite 'good' Interactive Learning material available, students prefer not to learn primarily using this approach. What Interactive Learning material is available, it is usually in stand-alone form without accompanying written material, which is a considerable drawback. This has been the Achilles heel of many Interactive Learning projects - failure to adequately link with the curriculum.

2. The Loughborough experience

At Loughborough we have put in place an environment for learning mathematics that we believe will be attractive to the vast majority of undergraduate students, whatever their level and whatever their previous experience.

In 1997, funding was made available by the University for the 'Open Learning Project' in Mathematics for Engineers which has provided high-quality student-centred workbooks, computer aided learning material closely allied to the workbooks, and a strategy for computer aided assessment which can be used for self-assessment and for module assessment. For students following this open learning regime, lectures are now optional as they can now choose to study, with guidance, the mathematics independently.

The success of the Open Learning Project encouraged staff to seek funding to develop further this work resulting in the HELM project (Helping Engineers Learn Mathematics) which was supported by a £250,000 HEFCE FDTL4 grant for the period Oct 2002-Sept 2005.

3. The HELM project

The HELM team comprised staff at Loughborough University and four consortium partners in other English universities: Hull, Manchester, Reading and Sunderland. The project's aims were to considerably enhance, extend and thoroughly test Loughborough's original Open Learning materials. These were to be achieved mainly by the writing of additional Workbooks and incorporating engineering examples and case studies closely related to the mathematics presented, enhancing the question databanks, upgrading the Interactive Learning segments and adding some more for basic mathematics topics, and promoting widespread trialling.

The HELM project's output consists of Workbooks, Interactive Learning segments, a Computer Aided Assessment regime which is used to help 'drive the student learning' and a report on possible modes of usage of this flexible material.

The Workbooks may be integrated into existing engineering degree programmes either by selecting isolated stand-alone units to complement other materials or by creating a complete scheme of work for a semester or year or two years by selecting from the large set of Workbooks available. These may be used to support lectures or for independent learning.

HELM's emphasis is on flexibility - the work can be undertaken as private study, distance learning or can be teacher-led, or a combination, according to the learning style and competence of the student and the approach of the particular lecturer.

4. HELM project Workbooks

50 Workbooks are available which comprise:

- 46 Student Workbooks (listed in 50.4) written specifically with the typical engineering student in mind containing mathematical and statistical topics, worked examples, tasks and related engineering examples.

- A Workbook containing an introduction to dimensional analysis, supplementary mathematical topics and physics case studies.

- A Workbook containing Engineering Case Studies ranging over many engineering disciplines.

- A Students' Guide

- A Tutor's Guide (this document)

The main project materials are the Workbooks which are subdivided into manageable Sections. As far as possible, each Section is designed to be a self-contained piece of work that can be attempted by the student in a few hours. In general, a whole Workbook typically represents 2 to 3 weeks' work. Each Workbook Section begins with statements of pre-requisites and the desired learning outcomes.

The Workbooks include (a) worked examples, (b) tasks for students to undertake with space for students to attempt the questions, and, often, intermediate results provided to guide them through problems in stages, and (c) exercises where normally only the answer is given.

It is often possible for the lecturer to select certain Sections from a Workbook and omit other Sections, possibly reducing the reproduction costs and, more importantly, better tailoring the materials to the needs of a specific group.

5. HELM project Interactive Learning Segments

The project has 80 Interactive Learning segments, which link to 22 of the lower level Workbooks. These enable web-based versions of the Workbooks to contain animations, which generate student interest. Revision exercises with randomly generated questions are provided for the benefit of students working independently. Web-delivered Interactive Learning segments complement the Workbooks and include audio, animation and self-assessment aspects. These have been found to be especially useful for supporting students of moderate mathematical ability, and for revision. They are also useful for illustrating lectures.

6. HELM project Assessment Regime

In formal educational environments assessment is normally an integral part of learning, and this is recognised by the HELM project. Students need encouragement and confirmation that progress is being made. The HELM assessment strategy is based on using Computer-Aided Assessment (CAA) to encourage self-assessment, which many students neglect, to verify that the appropriate skills have been learned. The project's philosophy is that assessment should be at the heart of any learning and teaching strategy and Loughborough University's own implementation of HELM makes extensive use of CAA to drive the students' learning.

HELM provides an integrated web-delivered CAA regime for both self-testing and formal assessment. HELM has around 5000 questions; most have a page of specific feedback.

7. HELM Transferability Project

Following on from the successful HELM Project in 2002-2005 a further HEFCE grant of £25,000 was awarded to encourage dissemination of good practice utilising HELM materials. This project ran October 2005 - July 2006 and involved the Loughborough HELM team working with six universities who were using (and adapting) HELM materials:

- Leicester

- Newcastle upon Tyne

- Nottingham

- Oxford Brookes

- Portsmouth

- Salford

Experiences of colleagues in the institutions have contributed to this Tutor's Guide.

HELM Consortium, Triallist Institutions and Individual Contributors

<div style="float:right;">**50.2**</div>

HELM learning resources have been produced primarily by a consortium of writers and developers at five universities:

Hull, Loughborough, Manchester, Reading, Sunderland.

The HELM consortium gratefully acknowledges the valuable support of many colleagues at their own and at the following institutions involved in additional writing, critical reading, trialling and revising of the learning materials

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Aston
Cambridge
City
Glamorgan
Glasgow
Glasgow Caledonian
Hertfordshire
Leicester
Liverpool
London Metropolitan
Newcastle
Northumbria
Nottingham
Nottingham Trent
Oxford Brookes
Plymouth
Queen's Belfast
Robert Gordon
Southampton
Southampton Solent
Surrey
Teesside
Ulster
University of Wales Institute Cardiff

Other HE/FE Institutions
Bournemouth & Poole College
Glenrothes Institute of Applied Technology
Harper Adams University College
Moray College
Royal Forest of Dean College
Sligo Institute of Technology
Westminster Kingsway College
West Notts College

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Contributors from other institutions

The following is a list of all those at other institutions who have helped in the development of the HELM materials by contributing examples and text, critically reading, providing feedback from trialling, pointing out errors, and offering general advice and guidance.

Felix Ampofof	Brian Lowe
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Sanowar Khan	James Vickers
Joseph Kyle	Juliette White
David Livie	James Wisdom
Peter Long	Hilary Wood
David Lowe	Lindsay Wood

HELM Transferability Project (HELMet)

1. Introduction

In September 2005 the original HELM FDTL4 project reached its successful conclusion culminating in a well attended conference at Loughborough University organised jointly by HEA-MSOR and HEA-Engineering. This marked the achievement of the original projects' intentions, namely to develop learning and teaching resources which would be instrumental in obtaining improved outcomes in mathematics for undergraduate engineers.

In 2005 transferability funding was obtained with the purpose of encouraging the effective transfer across institutions of the HELM learning and assessment resources. The resulting collaboration with Loughborough of six universities - Leicester, Newcastle, Nottingham, Oxford Brookes, Portsmouth and Salford - became the nucleus of the new one-year HELMet ("HELM educational transfer") project which ran October 2005 - July 2006.

It was anticipated that the partnership institutions would use the HELM deliverables in different pedagogic ways and that the project would report the difficulties, successes and failures in transferring HELM to these institutions, and thereby significantly enhance this HELM Tutor's Guide, and thus further aid the transferability process. Their reports are included in Section 50.5.

2. Partner Institutions and Individual Contributors

The following were involved in the HELMet project:

Loughborough University

Dr David Green

Dr Martin Harrison

Dr Aruna Palipana

David Pidcock

Helen Sherwood

Leicester University

Dr Phil Brown

Tony Corbett

Dr Da-Wei Gu

Dr Guido Herrmann

Dr Emmanuel Prempain

University of Newcastle-upon-Tyne

Dr Malcolm Farrow

Prof Robin Johnson

Dr Bill Foster

University of Nottingham

Dr Stephen Hibberd

Cliff Litton

Oxford Brookes University

Dr Robert Beale

Dr Richard Francis

University of Portsmouth

Dr Michael McCabe

Fiona Message

Lynn Pevy

Dr David Salt

Alison White

Salford University

Dr Ian Drumm

Dr Sabine von Hönerbein

Usage Modes of the HELM Resources

1. Introduction

During the course of the HELM Project some 30 institutions implemented one or more of the three aspects of the HELM Resources (Workbooks, Interactive Learning Segments, CAA questions) into at least one of their programmes. The implementations varied from simply making students aware that the materials existed and were available to them, to a full implementation of all three aspects of the resources entirely replacing materials previously in use.

Interviews were held with staff and individual students and focus groups were held at a number of trialling institutions over the course of the project. Staff also completed specially developed paper-based and electronic questionnaires and standard institutional student feedback forms were made available to the project for analysis. Emails were received from a number of staff (and a few students).

This report is based upon all such evaluative sources for the period 2002 - 2005. Furthermore, six institutions participated in the transferability phase in 2005-6 whose reports are presented in Section 50.5.

2. Summary of use of HELM resources

Of the 30 trialling institutions

- 100% made use of a selection of the 46 mathematics and statistics Workbooks then available

- 20% implemented the HELM CAA question bank

- 13% made the HELM Interactive Learning Segments available to their students.

3. Use of Workbooks

3.1 Workbooks as lecture notes (hard copy format)

In the cases where HELM Workbooks were used to replace existing lecture notes (20% of implementations), students were issued hard copies of the Workbooks relevant to their modules. Workbooks were either issued in full at the beginning of each topic or subsections were issued lecture by lecture.

The lecturers choosing to issue relevant sections lecture by lecture felt that this gave them the advantage of controlling what the students had before them in the class and allowing the lecturer to give very focussed direction to the students on what they should be doing during a particular time period. However, issuing complete Workbooks eased any complications in reproduction and simplified the situation when students had been absent, as they simply knew which Workbook to collect rather than having to identify particular subsections. In all cases, the materials were made available prior to the lecturer beginning the topic, allowing students to preview the material in advance of the lectures.

Lecturers, using the Workbooks as their core notes essentially employed one of two approaches to their lectures.

Some staff, particularly those teaching the material for the first time or for the first time to a particular group, developed lectures, often Powerpoint based, which reflected the content of the Workbooks exactly, using the same or very similar examples. Students found this reassuring; they felt safe that they did not have to take notes, therefore being able to focus on understanding, and they knew exactly where they were in the material enabling them to easily follow the lecture, and could make annotations to their Workbooks in the appropriate places. However, some students did feel that this made the lecture have less value as they could simply study for themselves at home and that in such lectures they did not have the opportunity to work through the examples themselves to reinforce their learning.

Other staff lectured in parallel to the content of the Workbook, using their own examples and developing some of the theory using their own existing notes. Students were kept appraised of which sections of the Workbooks were being addressed and were directed to work through specific examples and exercises to follow up the material covered within the lectures. Students appreciated that this allowed them to see more examples than just those contained in the Workbooks, but they did tend to feel that in these circumstances they needed to make more extensive notes. It is helpful to students that when this second approach is chosen that notation used by the lecturer is kept consistent with that in the Workbooks even where this is not the lecturer's preferred method - lecturers are more able to adapt than students struggling to comprehend new mathematical concepts.

3.2 Workbooks and CAA

In half of the cases where the HELM Workbooks have fully replaced existing notes, the HELM CAA questionbank has also been implemented to provide formative and/or summative assessments at the end of each topic. The students freely admitted that the level of effort they made using the Workbooks was proportional to the closeness of the next assessment. In all these trialling implementations the assessments were carried out online in a mix of supervised and unsupervised environments. The advantages and disadvantages of each environment are discussed later.

3.3 Workbooks as supplementary linked material for whole modules

The largest subgroup (50% of implementations) used the Workbooks as supplementary notes, linking them explicitly to the content of the lectures.

In 60% of these cases, hard copies of the Workbooks were issued to students at lectures or tutorials

thus providing an incentive to attend. In the other 40% of these cases, electronic links to the Workbooks sited on the institutions' websites were given.

Students were directed to use the material to help them understand the content of their lectures and to provide additional examples and exercises to use during tutorials and for private study. It was found that few students made the effort to work through the HELM material if they viewed it as an optional (albeit very useful) extra. Some students also expressed a lack of conviction about working through the Workbooks in class, being content to attend tutorials to collect the Workbooks and taking them home unopened.

Some success has been achieved using a peer tutoring scheme where the Workbooks are used as the subject matter. This provided a focus for the sessions and excellent support for students struggling with their mathematics and strong backup for their mentors.

Nevertheless, interviewed students did, in all cases, appreciate having the materials and felt that when they were preparing for examinations they would be very useful.

3.4 Workbooks as supplementary linked material for parts of modules

Of those institutions using the Workbooks as supplementary materials, around 20% restricted use of the materials to a small subset of the module content. They tended to use the most fundamental Workbooks (1: Basic Algebra, 2: Basic Functions, 3: Equations, Inequalities & Partial Fractions) as they wanted to establish the foundations with what would be regarded as fairly weak mathematicians. For more advanced mathematical topics they continued with their existing notes. They found that using the materials for this group allowed the students to be able to spend time honing their skills before moving on to more challenging topics.

3.5 Workbooks as unlinked supplementary material

Some 30% of implementations used the Workbooks as supplementary materials and made the Workbooks available to students but did not specifically attach them to particular modules or courses. In most cases the Workbooks were made available online for students to download and (if required) print off themselves. Links to the Workbooks were provided by lecturers and/or through support centres.

In the support centres, hard copies were sometimes obtainable on request or at least there were reference copies available. It is difficult to measure the rate of uptake in these circumstances. However, having this range of materials available as part of a support service for students is seen as valuable by students and staff alike.

It was noticeable that **mature students** and those with **special needs** (typically dyslexics) were **very appreciative** of the Workbooks and did make extensive use of them, no matter which usage mode was employed, citing being able to work at their own pace and being confident that they had a complete set of notes as large positives.

3.6 Tutorial assistants

Many institutions use postgraduates for tutoring and the HELM Workbooks provide a sure foundation for them. (It is well documented that staff often fail to keep their tutorial assistants adequately informed of their lecture notes and chosen methods, and students seeking help often have woefully inaccurate notes.)

4. CAA issues

- Students feel less pressured when unsupervised.

- It is simpler to schedule unsupervised assessments as rooms and staff do not need to be booked at specific times.

- The incidence of plagiarism in unsupervised testing cannot be readily measured and appears to be of growing concern.

- Some students reported that they felt disadvantaged if they worked alone knowing that other students colluded.

- To address the issue of plagiarism some lecturers have implemented a mixture of supervised and unsupervised testing. Where a computer lab is unavailable, the supervised testing has been carried out in class and marked by hand. By comparing performances in the two environments, this has provided evidence of whether or not students are colluding and so far the results indicate that plagiarism is probably not as large a problem as perceived.

5. Interactive Learning Segments

By its very nature the Interactive Learning material was used in electronic format and so usage has been hard to quantify. Some students have expressed great enthusiasm for it whereas others have found it too laborious or slow paced. It seems to have been best received and used by mature students.

6. Conclusions

The HELM materials have been used effectively in a variety of modes and combinations.

It is expected that uptake will continue to grow once the final product is widely disseminated. There is some evidence that the HELM materials will spread within an institution once being used.

A HEFCE-funded transferability study undertaken Oct 2005-July 2006 has provided further and better information about efficacious modes of use as demonstrated by the six reports in the following section.

HELM Uptake and Transferability Project Reports

1. Uptake of HELM materials in general

By September 2005 the HELM project had 49 registered participants covering 41 institutions split between HE (35) and FE(HE) (6), of which 35 were in England and N.I.

Though funded by HEFCE principally for the benefit of those institutions in England and Northern Ireland, HELM attracted interest from other regions as the following table illustrates.

HELM uptake by Region

For the period following the project's conclusion in September 2005 to June 2006 a further 22 new institutions were added to the list of registered users of the project materials, two being in the FE(HE) sector.

The updated regional distribution as at June 2006 was:

Uptake by Region Jun 06

Thus the total number of institutions in the UK and Eire known to be using HELM in June 2006 had risen to 63 (HE: 55, FE(HE): 8). Of these, 47 were in England and N.I. (an increase of 16 over September 2005).

As one would expect, the majority of institutional requests for HELM materials come from members of Engineering Departments or Mathematics Departments where there is a teaching or supporting role for engineering students. However, there is a degree of diversity: for example we have also received requests from a Medical Statistician teaching Statistics to Medical students and from a Chemistry Lecturer responsible for providing Mathematics tutorials to Chemistry students. In a wider context our materials are used by the Open University Faculty of Technology and are available to OU students and staff from selected Technology course websites. We also find our material has been requested for possible use in a wider supportive role such as that provided by the Skills Centre based at Leeds University (http://www.leeds.ac.uk/skillscentre).

While our materials were primarily designed for use within the HE sector, we also have recorded some interest from the FE sector where our coverage of topics on Algebra, Algebraic Fractions, Differentiation and Integration has been found particularly useful.

2. The HELM transferability project

In the period October 2005 July 2006 transferability funding from HEFCE-HEA was obtained with the purpose of encouraging the effective transfer across institutions of the HELM learning and assessment resources. The resulting collaboration of the universities of Leicester, Newcastle, Nottingham, Oxford Brookes, Portsmouth and Salford constituted the HELMet ("HELM educational transfer") project.

The following six extensive reports from the six participating universities provide a rich variety of insights into uses of HELM materials and developments thereof.

3. The use of HELM at Leicester University

Contributors: Dr Da-Wei Gu, Tony Corbett, Dr Emmanuel Prempain, Dr Guido Herrmann, Dr Phil Brown

Institution Background

The University of Leicester is a medium size, research university in the Midlands. The Department of Engineering is a general engineering department and offers a range of degree courses at BEng/MEng level, including Communications & Electronic Engineering, Electrical & Electronic Engineering, Electronic & Software Engineering, Mechanical Engineering and General Engineering. There are about 100 students in each years cohort. Three mathematics modules are taught: two in the first year course (one for each semester) and the third one in Semester 1 of the second year. All students are required to attend those modules which are given by the staff members of the Department.

Use of HELM Products

We started to use HELM workbooks and introduced Question Mark Perception as a continually assessed element into our first year maths teaching in 2003-04. In 2005-06, we further recommended HELM workbooks in the second year maths teaching. Sixteen workbooks (Nos. 6, 7, 9-14, 16, 18-20, 23, 35-38) are used in the first year and 6 (Nos. 22, 24, 25, 27-29) in the second year. Those workbooks cover most of the syllabuses of the first year mathematics modules (EG1010 and EG1070) and part of the second years (EG2010). Purchase of the workbooks is compulsory. The mathematics teaching in the first year is based on the HELM workbooks while the workbooks in the second year are just recommended, reference material.

In the first year's course, we set 6 assessments in each semester using QM Perception software (CAA). For each assessment, the students have 4 days for practice. During this period, students can make any number of attempts at the questions and see feedback when it is available. That is followed by a two-day period for the formal assessment. For the formal assessment, two hours are (generously) allowed and only the first attempt is counted. The formal assessment is not a timetabled session. Instead, the students can attempt it anytime in the 48-hour period over the internet where a computer on campus or in hall can be used. The resultant marks of all 6 assessments are counted as 40% of the module mark. The remaining 60% is from a written paper exam at the end of the semester. The table below lists the average marks for the past 6 years, before and after the use of HELM,

	EG1010 (Mathematics 1)		EG1070 (Mathematics 2)	
	Written Examination	CAA tests	Written Examination	CAA tests
2000-01	54.1%	n/a	51.1%	n/a
2001-02	51.8%	n/a	48.3%	n/a
2002-03	56.5%	n/a	51.6%	n/a
2003-04	58.3%	57.6%	60.8%	64.2%
2004-05	55.4%	51.6%	51.9%	68.1%
2005-06	63.5%	62.7%	50.6%	68.1%

It is encouraging to see that the written paper exam mark of EG1010 in the last year (2005-6) has improved significantly.

Feedback from Students

The general student feedback on HELM materials (Workbooks and CAA) is positive. They found these aids very useful. In particular, they like the following features:

- Easy to understand

- Very informative

- Large number of questions at different levels

- Computerised learning and assessing

- Clear layout/organisation of material, though some want to reduce the overlap between work-books

A few students would like to see more challenging, comprehensive questions.

Feedback from Staff

Five staff members are involved with teaching, setting up QM Perception exams and maintaining the software. Again, the general response is very favourable. We think the HELM workbooks are well designed and compiled. They are a very useful tool to assist the maths teaching. We appreciate the number of available questions with worked-out solution procedures which greatly help students in developing their self-learning and problem-solving skills. For engineering students in particular, the workbooks and CAA have been designed to focus on problem solving rather on theories.

The workbooks helped us in preparing teaching material and, thus, reduced our workload. Tutorial time has been reduced as well, since students can use Blackboard to communicate the lecturers about their queries and get the response.

We would like to see more materials on vectors, linear algebra, 2D/3D geometry, for instance, which are all important for engineering students.

Setting up CAA sessions and maintenance of Perception are straight forward. It is also easy to arrange individual schedules for students who are ill or have their sessions cut off in the middle of a test. The facility to export summary CSV file is very helpful. We created a script file to extract the information details we are interested in.

Summary

In brief, we think HELM Project and the products it generated are a great support to mathematics teaching for engineering students such as ours. It can raise teaching quality, improve students' learning, as well as reduce staff workload. We will continue to use the workbooks and CAA software, and are considering extending the CAA into the second maths module.

4. Videos for HELM at the University of Newcastle

Contributors: Dr Bill Foster, Prof Robin Johnson

1. Introduction

Past experience at the University Newcastle upon Tyne

For some time, short videos have been used at Newcastle to support some mathematics and statistics service modules for engineers and scientists. The focus has been on solving typical exercises from the modules. Students have accessed these via a VLE or a CD. The videos have been used as back-up resource and have been welcomed by students as being very useful. This was especially so in a large engineering class this academic year. The videos used hitherto were clips produced by chopping up those created in the Mathtutor project.

The HELM project

The opportunity arose via the HELM project to follow up this experience. The idea was to exploit the structure of the HELM Workbooks, with their standard approach to engineering mathematics and statistics, together with the worked examples and exercises, to create appropriate videos for those examples and exercises. During the transferability project phase 80 videos have been created. The initial focus has been on the basic mathematics found in the early Workbooks and on some elementary probability and statistics. These videos will be linked from the electronic Workbooks, streamed from the Newcastle or any appropriate site, or can be supplied on CD Roms or DVD.

Video Quality and Compromises

The videos have been looked at by about 20 students finishing their first year at Newcastle and the feedback (limited and not part of a module) has been positive, both in terms of the perceived usefulness and the quality of the videos. The videos are not of high production standard but are seen as reasonable by all reviewers so far; lighting could be improved and there is often some external noise. This did not seem an issue with the students. This extraneous noise was a consequence of using any reasonable and available room for videoing, often at short notice. The original proposal was for 20 videos, but we quickly found that the pragmatic approach adopted fitting in with staff commitments, choosing a fixed format for videoing, choosing a standard and easy to use editing tool, having postgraduates who were interested and capable, being able to set up equipment very quickly - led to an easy-to-manage process with much quicker turn around than expected and so we have produced four times as many videos as originally planned.

2. Summary of videos produced

80 videos, each up to a maximum of 5 minutes in length.

- The videos can be linked to/from the relevant HELM pdf files.

- The videos can be used stand-alone.

- The videos are accessible from Newcastle by any course developer in an HE institution.

- The content and topics are based on the material in the HELM workbooks and agreed by HELM.

- The format is semi-formal, with a lecturer speaking directly to the camera and writing on a white-board.

- On average, a video took about 15 minutes to video, and 1 hour to edit and produce a final version suitable for publication.

- A final year postgraduate student was employed for the videoing and editing.

3. Why Videos?

There is evidence that short videos of experienced presenters working through the solutions of standard exercises and problems in basic mathematics and statistics are of value in reinforcing student acquisition of important skills. One recent project, Mathtutor, produced videos of a relatively high standard together with careful indexing. These are part of CD Roms on elementary engineering mathematics with theory, practice, worked examples and quizzes all indexed and cross referenced.

For Mathtutor see: http://www.mathtutor.ac.uk/

Also see the mathcentre site:

http://www.mathcentre.ac.uk/resources_for_category.php?f=1&c=32

where these videos are also linked.

However, students appreciate their initial exposure to new mathematics or statistics from lectures and/or tutorials as human interaction is seen to be important at this stage. A good tutor/lecturer can quickly assess the students background knowledge and tailor the explanation and presentation according to the learning styles of the students; this also gives the students greater confidence. Once the initial learning stage is over, then true facility in basic mathematical technique is achieved by reinforcement and repetition via plenty of practice. It is at this stage that videos are seen as useful giving plenty of opportunity to review the material in the video with immediate feedback subject to student control. It is still very important to have tutor help available as the videos solutions may not cover every difficulty that a student may encounter in solving similar exercises; it is wise to regard. at this point, the videos as back-up material rather than central to the practice regime.

At Newcastle, during the last three years, we have used the Mathtutor videos on Blackboard modules for several service courses in mathematics and statistics for Computer Scientists, Physics and Chemistry students, and for Engineering students of all disciplines.

Initially we had to break the videos, after permission, into suitable smaller clips; each of these clips covered an exercise on a skill. We found that there were two factors which determined the student use of the videos:

1. The length of the video 2. The relevance of the video.

The video had to be short and directly relevant to the task that the student was undertaking : for example, assignments (whether CAA or text based, module exercises), for aiding the understanding

a text based solution or for practising a past-exam. The video had to be precisely aimed at the problem that the student needed help on, or else it was seen as irrelevant. Also the video should be easily reached usually only a link away. Feedback from students on the directed use of such videos was very positive. This was especially so for the large engineering class; there was especially positive feedback from the dyslexic students. The evidence is that videos do increase student engagement as measured by the amount of time they spend in looking at and using the various available resources. The drawback to this is that many short videos need to be created and, at first sight this seems to be a major resource problem; quality issues to do with the presentation of the video may also become important. Those issues are dealt with later, but first we address the problem of the method of presentation.

4. Video Presentation: Emulating the tutorial

One approach is to emulate as closely as possible the learning environment that most students are familiar with and consistently report as the most useful: the small group tutorial with a tutor going through examples on the board with full explanations . Ideally student/tutor interaction should be present and the original proposal to HELM had this as a possible format but this was not feasible for resource and time constraints (and not least being able to guarantee an audience). The presenter of the video should speak clearly, write legibly, be enthusiastic and behave as if there is engagement with a student by addressing the camera. Not too much material should be written on the board as moving the board up and down can be distracting. One principle we adopted is that there should be minimal camera movement or zooming - for short videos this is not necessary and there is the added advantage of a fixed camera position so no real expertise is needed by the camera operator. Each video should be as short as possible, consistent with a satisfactory and correct solution and full explanation of a worked example. If the video is too long then students tend to switch off and/or may be intimidated at the amount of detail implied by the length of the video. There is no need at this level of mathematics to have long explanations. The advantage of videos is that students can look at them as many times as they like, and will do so if they are short and relevant.

5. Creating the Video practical issues

Our experience is that videos need not be of a high quality as long as the presenter does a good, clear job of explaining. Although we have had limited student feedback on the videos we have produced, all of this has been positive there were no quality issues raised and the videos were perceived as very useful.

The video session

Lighting and sound are of course important however we did not use a studio as we required access to rooms at short notice; we found that a standard lecture room had adequate lighting and, as long as the camera was not too far from the lecturer, the sound picked up by internal microphone of the video camera was also adequate. Typically we would use a room without too much external noise and for which all lights were working, and set up the camera on a tripod facing a not-too-dirty whiteboard about 6-8 metres away. The presenter would prepare for each video by writing any titles and other appropriate material on the whiteboard before videoing. The presenter would then use a black pen and speak to the camera and would control when to start, stop and when to repeat (if necessary). The camera operator would usually be a postgraduate student whose job was to start and stop the camera and to listen for any obvious mistakes and then interrupt, if appropriate. The

camera operator would also provide the speaker with an audience.

Editing

Given the time constraints, and the number of videos created we decided to produce all videos on Windows media format (.wmv) and use Windows MovieMaker to edit although we have kept all AVI files in order to produce RealPlayer files later. This kept editing time down to a minimum and a short video of say 4 minutes would take about 1 hour in all. Postgraduate students did most of the editing with an overview by the presenter. Most videos were selected to run at 768 Kbs as this produced suitable quality, given either access via CD Rom or DVD or by broadband.

Equipment

- Sony DCR PC1000E DV camcorder

- spare battery

- tripod

- ten dv60 tapes.
     Cost: around 850 ex VAT.
  Ideally:

- external microphone

- extra lighting.
     Cost: around another 100.

Software

- Windows Movie Maker

- Adobe Premier Elements 2.0

- RealProducer Plus

- Adobe Acrobat.
     Cost: 200-300

Personnel and required training

- Presenters: Members of staff, willing to be minimally trained in presenting. Training consists of adopting a natural enthusiastic style as in a small tutorial, wearing appropriately coloured clothes, not moving around too much, not obscuring the board, writing legibly, talking clearly at an appropriate speed and not making too many mistakes!

- Camera Operators: Postgraduates. Training is minimal given the format of a more or less fixed camera.

- Editors: Postgraduates. Once again, the shortness of the videos and the minimal format makes editing via Movie Maker, Adobe Premier or RealProducer relatively straightforward. The two postgraduates used at Newcastle for editing found this easy to do once the basic production process was agreed. Training was about one or two hours of demonstration.

Production Control

Quality Control. The Video Preparation Record (see appendix), usually completed by the postgraduate editor, was the means by which the production process was controlled, timed and monitored. The finished video was then uploaded to the streaming server.

Production Costs

(After all equipment costs).

Payment to postgraduates for videoing and editing: 12.00 an hour. The only other major resources used were rooms (free), and staff time (free).

Staff time:

1. Preparation time per video: looking at a workbook and preparing an example from a section.

2. Presenting. No video took more than 20 minutes in presentation (including repeats)

3. Reviewing. About half an hour per video.

Postgraduate time:

1. Videoing: 15mins per video average.

2. Editing and other post processing: One hour per video. Sometimes longer if, after review, changes were needed.

6. Dissemination

1. All videos are on the Newcastle streaming server stream.ncl.ac.uk running at least at 768 Kbits/s.

2. On CD Roms and DVD.

Linking from HELM workbooks

It is easy to install links into the present pdf files using Adobe Acrobat. This is a standard editing process.

7. List of Video-clips for Sections in HELM Workbooks

Workbook 1 Basic Algebra		11 videos
1.1	Mathematical Notation and Symbols	*Not covered*
1.2	Indices	Index laws; Negative indices;
1.3	Simplification and Factorisation	Expand the difference of squares; Another expansion; expansion of three brackets; Factorising a complete square; Factorisation example 1; Factorisation example 2; Factorisation example 3;
1.4	Arithmetic of Algebraic Fractions	Combining algebraic fractions;
1.5	Formulae and Transposition	Changing the subject of an equation (2 videos)
Workbook 2 Basic Functions		16 videos
2.1	Basic Concepts of Functions	*Not covered*
2.2	Graphs of Functions and Parametric Form	Comparing curves; Sketching a function specified in 2 parts; Solving equations using graphs
2.3	One-to-one and Inverse Functions	Checking that a function is one-to-one; Finding the inverse of a function.
2.4	Characterising Functions	Sketching a function; Periodic functions
2.5	The Straight Line	Equation of a line through two points; (not yet included) Equaton of a line given the gradient and one point; (not yet included)
2.6	The Circle	The equation of a circle;
2.7	Some Common Functions	Sketching a function involving the modulus; Graphing sine of a multiple angle; Periodic functions; Sketching $x + 1/x$; Sketching parabolae; Sketching $\tan(x)$

Workbook 3 Equations, Inequalities and Partial Fractions		**14 videos**
3.1	Solving Linear Equations	Solving a linear equation; solving a rational equation
3.2	Solving Quadratic Equations	Solve complete square; factorisation2;
3.3	Solving Polynomial Equations	Roots cubic;
3.4	Solving Simultaneous Linear Equations	Solving a pair of linear eqns;
3.5	Solving Inequalities	Linear inequalities;
3.6	Partial Fractions (some to be redone)	Partial fractions introduction; Linear factors in denominator 1; Linear factors in denominator 2; Comparing coefficients; Quadratic factor in denominator 1; Quadratic factor in denominator 2; Repeated linear factors;
Workbook 4 Trigonometry		**2 videos**
4.1	Right-angled Triangles	Trig pythag1(finding the side of a triangle);
4.2	Trigonometric Functions	(Covered elsewhere)
4.3	Trigonometric Identities	Expand $\cos 6x$;
4.4	Applications of Trigonometry to Triangles	*Not covered*
Workbook 6 Exponential and Logarithmic Functions		**7 videos**
6.1	The Exponential Function	Simplifying powers;
6.2	The Hyperbolic Functions	Formulae for hyperbolic functions;
6.3	Logarithms	Graphing by taking logs of a function; Simplifying log expressions;
6.4	The Logarithmic Function	Solving an equation using logs; Solving an equation involving exponentials; Solving the hyperbolic equation $\sinh(4x) = 1$;
6.5	Modelling Exercises	*Not covered*
6.6	Log-linear Graphs	*Not covered*

Workbook 11 Differentiation			**14 videos**
11.1	Introducing Differentiation	Differentiating from first principles (an elementary function); Differentiating from first principles; the square root function;	
11.2	Using a Table of Derivatives	Differentiating standard functions;	
11.3	Higher Derivatives	First and second derivatives;	
11.4	Differentiating Products and Quotients	Differentiate product of functions; Differentiate product of 3 functions; Differentiate using the quotient rule (2 videos);	
11.5	The Chain Rule	Diff chain rule1;	
11.6	Parametric Differentiation	Parametric differentiation; Parametric differentiation - first and second derivatives;	
11.7	Implicit Differentiation	Implicit differentiation ; Implicit diff $2(\arctan(x))$; Inverting the second derivative;	
Workbook 12 Applications of Differentiation			**7 videos**
12.1	Tangents and Normals	Implicit differentiation - finding the tangent and normal to a surface;	
12.2	Maxima and Minima	Maximum example (practical) Minimum example (practical)	
12.3	The Newton-Raphson Method	*Not covered*	
12.4	Curvature	*Not covered*	
12.5	Differentiation of Vectors	Differentiation of a vector and scalar times a vector; Differentiation scalar product; Rotating frames; Velocity and acceleration in a moving frame;	
12.6	Case Study: Complex Impedance	*Not covered*	

Workbook 13 Integration		15 videos
13.1	Basic Concepts of Integration	Two standard integrals; Integration of $f'(ax + b)$; Integration of a function of a function;
13.2	Definite Integrals	Three definite integrals;
13.3	The Area Bounded by a Curve	Area under a curve; Bounded area between curves;
13.4	Integration by Parts	Integration by parts1 Integration by parts2; Integration by parts twice; Integration by parts leading to an equation for the integral
13.5	Integration by Substitution Using Partial Fractions	Indefinite integral using substitution; Definite integral using trip substitution Indefinite integral using partial fractions;
13.6	Integration of Trigonometric Functions	Indefinite integral using trip identities; Reduction formula for integrating powers of $\tan(x)$; (to be redone)
Workbook 35 Sets and Probability		2 videos
35.1	Sets	*Not covered*
35.2	Elementary Probability	*Not covered*
35.3	Addition and Multiplication Laws of Probability	*Not covered*
35.4	Total Probability and Bayes' Theorem	The law of total probability; Bayes Theorem;
Workbook 37 Discrete Probability Distributions		2 videos
37.1	Discrete Probability Distributions	*Not covered*
37.2	The Binomial Distribution	An example using the Binomial distribution;
37.3	The Poisson Distribution	Poisson Process (an example);
37.4	The Hypergeometric Distribution	*Not covered*
Workbook 39 The Normal Distribution		1 video
39.1	The Normal Distribution	The normal distribution;
39.2	The Normal Approximation to the Binomial Distribution	*Not covered*

8. List of Video-clip web links

Title	HELM Reference	web link
Index Laws	1.2	mms://stream.ncl.ac.uk/w.h.foster/Jim/ 1.2/JWMF_index_laws.wmv
Index Laws - negative indices	1.2	mms://stream.ncl.ac.uk/w.h.foster/Jim/ 1.2/JWMF_index_negative.wmv
Expansion of brackets (three)	1.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 1.3/RSJ_expansion.wmv
Factorisation - Completing the square	1.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 1.3/RSJ_factor_complete_square.wmv
Factorisation	1.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 1.3/RSJ_factorisation.wmv
Expansion of Brackets - the difference of two squares	1.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 1.3/RSJ_expand_diff_squares.wmv
Solving a quadratic using factorisation 2	1.3	mms://stream.ncl.ac.uk/w.h.foster/Jim/ 1.3/JWMF_factorisation2.wmv
Solving a quadratic using factorisation 1	1.3	mms://stream.ncl.ac.uk/w.h.foster/Jim/ 1.3/JWMF_factorisation.wmv
Combining Algebraic Fractions	1.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 1.4/RSJ_combine_alg_frac.wmv
Changing the subject of an equation 1	1.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/ May/1.5/RSJ_change_subject_equation.wmv
Changing the subject of an equation 2	1.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/ May/1.5/RSJ_change_subject_equation2.wmv
Sketching a function specified in two parts	2.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.2/RSJ_sketch_function.wmv
Solving an equation using graphs	2.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.2/RSJ_solving_equation_graphs.wmv
Comparison of Curves	2.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.2/RSJ_compare_curves.wmv
Checking that a function is one-to-one	2.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.3/RSJ_one_to_one_function.wmv
Finding the inverse of a function	2.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.3/RSJ_inverse_function.wmv
The equation of a circle	2.6	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.6/RSJ_equation_circle.wmv
Sketching a function involving the modulus	2.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.7/RSJ_sketch_modulus.w
Sketch of a parabola	2.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.7/RSJ_sketch_parabola.wmv
Sketching graphs 1: x+1/x	2.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.7/RSJ_sketch_graph1.wmv

Graphing sine of a multiple angle	2.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.7/RSJ_sin_multiple_angle.wmv
Periodic Functions(version 2)	2.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.7/RSJ_periodic_functions2.wmv
Sketch of the function tan(x)	2.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 2.7/RSJ_sketch_tan.wmv
Solving a rational equation	3.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 3.1/RSJ_solv_rational_equation.wmv
Solving a linear equation	3.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 3.1/RSJ_solve_linear_equation.wmv
Solving a quadratic equation - completing the square	3.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 3.2/RSJ_solve_complete_square.wmv
Roots of a cubic - given one root	3.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 3.3/RSJ_roots_cubic.wmv
Solving a pair of equations	3.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 3.4/RSJ_solving_pair_lineareqns.wmv
Solving a linear inequality	3.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 3.5/RSJ_linear_ineq.wmv
Partial Fractions: Repeated Linear Factors	3.6	mms://stream.ncl.ac.uk/w.h.foster/Bill/ 3.6/WHF_parfrac_repeated.wmv
Partial Fractions: Quadratic Factors	3.6	mms://stream.ncl.ac.uk/w.h.foster/Bill/ 3.6/WHF_parfrac_quadratic.wmv
Partial Fractions: Comparing coefficients	3.6	mms://stream.ncl.ac.uk/w.h.foster/Bill/ 3.6/WHF_parfrac_compare_coeff.wmv
Partial Fractions: Introduction	3.6	mms://stream.ncl.ac.uk/w.h.foster/Bill/ 3.6/WHF_parfrac_intro.wmv
Partial Fractions: Linear Factors	3.6	mms://stream.ncl.ac.uk/w.h.foster/Bill/ 3.6/WHF_parfrac_linear.wmv
Partial Fractions: Linear Factors(2)	3.6	mms://stream.ncl.ac.uk/w.h.foster/Bill/ 3.6/WHF_parfrac_linear2.wmv
Partial Fractions: Quadratic Factors(2)	3.6	mms://stream.ncl.ac.uk/w.h.foster/Bill/ 3.6/WHF_parfrac_quadratic2.wmv
Using Pythagoras' Theorem to find the side of a triangle	4.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 4.1/RSJ_trig_pythag1.wmv
Expansion of cos(6x)	4.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 4.3/RSJ_trig_expand_cos6x.wmv
Simplifying powers	6.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 6.1/RSJ_Simplifying_powers.wmv
Formulae for hyperbolic functions: sinh(2x), cosh(2x) and tanh(2x)	6.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 6.2/RSJ_formulae_hyperbolic_2x.wmv
Simplifying log expressions	6.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 6.3/RSJ_simplify_log_expressions.wmv
Graphing by taking logs of a function	6.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 6.3/RSJ_graphing_taking_logs.wmv
Solving an equation using logs	6.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 6.4/RSJ_solving_equation_taking_logs.wmv

Solving an equation involving exponentials	6.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 6.4/RSJ_solve_equation_exponentials.wmv
Solving the hyperbolic equation sinh(4x)=-1	6.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 6.4/RSJ_solve_hyperbolic_equation1.wmv
Differentiating from first principles	11.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.1/RSJ_diff_first_principles.wmv
Differentiating from first principles: the square root function	11.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/11.1/ RSJ_diff_first_principles_continued_squareroot.wmv
Differentiating standard functions: an example	11.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.2/RSJ_diff_standard_functions_example.wmv
First and second derivatives	11.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.3/RSJ_first_and_second_derivatives.wmv
Differentiation: Product of three functions	11.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.4/RSJ_diff_prod_3functions.wmv
Differentiation: The Quotient Rule	11.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.4/RSJ_diff_qurule1.wmv
Differentiation: The Product Rule	11.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.4/RSJ_diff_pr1.wmv
Differentiation of a Quotient using Product and Chain Rules	11.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.4/RSJ_diff_quotient2.wmv
Differentiation: The Chain Rule for a function of a function	11.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.5/RSJ_diff_chainrule1.wmv
Parametric Differentiation 2: First and second derivatives	11.6	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.6/RSJ_parametric_diff2_second_deriv.wmv
Parametric Differentiation	11.6	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.6/RSJ_parametric_diff.wmv
Implicit Differentiation 2: derivative of arctan(x)	11.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.7/RSJ_implicit_diff2_arctan(x).wmv
Implicit Differentation 1	11.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.7/RSJ_implicit_diff1.wmv
Differentiation: Inverting the second derivative	11.7	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 11.7/RSJ_diff_invert_second_deriv.wmv
Implicit Differentiation 3: tangent and normal to a curve	12.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 12.1/RSJ_implicit_diff3_tangent_normal.wmv
Differentiation: Maximising a function. A Practical Example	12.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 12.2/RSJ_diff_max_example.wmv
Differentiation: Minimising a function. A practical example	12.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 12.2/RSJ_diff_min_example.wmv
Differentiating scalar times a vector	12.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 12.5/RSJ_diffvector_scalartimesvector.wmv
Differentiating a scalar product of vectors	12.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/ 12.5/RSJ_diffvectors_scalarproduct.wmv

Rotating Frames	12.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/12.5/RSJ_rotatingframes.wmv
Velocity and Acceleration in a moving frame	12.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/12.5/RSJ_vel_acc_movingframe.wmv
Two indefinite integrals using standard derivatives	13.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.1/RSJ_two_standard_integrals.wmv
Integration example: Function of a function	13.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.1/RSJ_int_functionofafunction1.wmv
Integrating f'(ax+b)	13.1	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.1/RSJ_int_f(ax+b).wmv
Three Definite Integrals	13.2	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.2/RSJ_three_definite_integrals.wmv
Definite Integration: Area under a curve	13.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.3/RSJ_area_under_curve.wmv
Definite Integration: Bounded area between two curve	13.3	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.3/RSJ_area_between_curves.wmv
Indefinite Integration: Integrating by Parts 1	13.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.4/RSJ_int_by_parts1.wmv
Indefinite Integration: Integration by Parts 2	13.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.4/RSJ_int_by_parts2.wmv
Definite Integration: Integrating by Parts Twice	13.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.4/RSJ_int_by_parts_twice.wmv
Definite Integration: Integrating by Parts Twice(2)	13.4	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.4/RSJ_def_int_bypartstwice(2).wmv
Indefinite Integration: Substitution 1	13.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.5/RSJ_indef_int_substitution1.wmv
Definite Integration: A trig substitution	13.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.5/RSJ_definite_integration_trig_substitution.wmv
Indefinite Integration: Partial Fractions 1	13.5	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.5/RSJ_indefinite_int_partial_fractions1.wmv
Indefinite Integration: Using Trig Identities	13.6	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.6/RSJ_indefinite_int_using_trig_identities.wmv
Reduction formula for powers of tan(x)	13.6	mms://stream.ncl.ac.uk/w.h.foster/Robin/Integration/13.6/RSJ_reduction_formula_tan(notitle).wmv
The law of total probability	35.4	mms://stream.ncl.ac.uk/w.h.foster/Phil/35.4/PSA_total_prob.wmv
Bayes' Theorem	35.4	mms://stream.ncl.ac.uk/w.h.foster/Phil/35.4/PSA_bayes_thm.wmv
An example using the binomial distribution	37.2	mms://stream.ncl.ac.uk/w.h.foster/Phil/37.2/PSA_Binomial_Distribution1.wmv
Poisson Process: An Example	37.3	mms://stream.ncl.ac.uk/w.h.foster/Phil/37.3/PSA_Poisson_Process1.wmv
Poisson Process: An Example(1)	37.3	mms://stream.ncl.ac.uk/w.h.foster/Phil/37.3/PSA_Poisson_Process_Example1.wmv
The normal distribution	39.1	mms://stream.ncl.ac.uk/w.h.foster/39.1/Malcolm/MF_normal.wmv

Appendix: Video Preparation Record

Video Record	
Presenter	
Title	
Date and Time	
Room Comments on Room	
Camera Operator	
Duration of Videoing	
Tape Record	
Tape Number	
Position on Tape	
Duration of video on tape	
Editing Record	
Wmv files	
AVI file created and saved	
wmv file created and saved	
Name	
Location	
Bit rates	
Titles added	
Size of file	
Reviewed and passed	
Location	
Streaming file created	
Location	
Editor	
Time taken	
RealPlayer files (.rm or .rv)	
RealPlayer file created and saved	
Name	
Bit rates	
Titles added	
Size of file	
Reviewed and passed	
Location	
Streaming file created	
Location	
Editor	
Time taken	

5. The use of HELM at the University of Nottingham

Contributor: Dr Stephen Hibberd

Introduction

The provision of a HEFCE Transferability award to the FDTL4-HELM project enabled HELM materials to be embedded within the Service Mathematics provision at Nottingham University to enhance support for the learning of mathematics to Engineering and Science undergraduate students. HELM materials are now readily available to students for general study support in Mathematics and module specific support within a local VLE provision (MELEES - Mathematics Electronic Learning Environment in Engineering and Science). This report identifies the embedding of HELM workbooks into MELEES and considers of CAA for formative and summative assessment.

1. MELEES Mathematics Electronic Learning Environment in Science and Engineering

The principal challenge within the service teaching of mathematics is to address the needs of students who are often not provided with as extensive a support structure as that associated with their main subject area(s). The scale and diversity of the service teaching provision at Nottingham invited the establishment of a web-based development focussing on:

- Establishing a unified supportive environment matched to the student intake and needs;

- Providing learning and assessment resources to students from established local expertise;

- Incorporating additional supportive materials (e.g. from FDTL and LTSN initiatives);

- Identifying, informing and supporting e-learning strategies for module staff;

- Enabling specialist support for students with special learning needs.

MELEES now provides such an integrated framework for supporting students in learning mathematics. Students are given access to module materials strategically released in concert with the module teaching. Postgraduate Student Teachers and client School staff have prior access to student learning materials to help in planning and providing tutorial support.

MELEES is based on WebCT and when introduced in 2003 was in the vanguard with respect to implementation issues and student experiences gained from integrated use of a VLE. The enactment of the Special Educational Needs and Disability Act (SENDA) has highlighted the responsibility of teachers in providing accessible resources. Special attention within the MELEES project has been made to identify the extremely supportive role e-learning can offer.

Students have enthusiastically adopted using MELEES as an effective way of obtaining quality learning support. It has also been well used by many teaching staff as a means of providing an integrated and versatile support mechanism. This is enabled within a VLE by proving flexibility in terms of access both in terms of location and time and student choice. Students are using the environment extensively, this in turn encourages a culture to support and invest in more 'e-learning' activities.

Registrations onto MELEES in 2004-5 expanded to the whole service teaching cohort of some 1500 students, with regular use made by over 83% of the cohort. A telling result is that only 4% of students discarded MELEES after an initial login. The extent of MELEES usage to support learning was explored between the end of teaching in Autumn 2005 and the end of the semester. An email survey of students (130 replies) identified 88% used MELEES within that period; with graded responses 1. (extremely useful) 36%, 2. 37%, 3. 11%, 4. 13%, 5. (not useful) 3%. Associated locations used were: Campus 58%, Nottingham (off campus) 62%, Outside Nottingham (UK) - 52%, Outside Nottingham (Overseas) 3%.

MELEES has significantly influenced the development of e-learning in Mathematical Sciences at Nottingham and it has become embedded and integrated into the Schools teaching and learning provision. A more detailed overview on the development of MELEES is provided in [1].

Website: www.maths.nottingham.ac.uk/melees (info entry: username/password: melees2006)

2. Embedding HELM Workbooks

The increasingly available technology associated with 'e-learning' is now predominantly web-based and a growing implementation within HE Institutions of Virtual Learning Environments (VLEs). Such environments, with an underpinning content management system (CMS), enable learning and support materials to be configured, targeted and readily managed to local requirements by teaching staff and readily link to other local or possibly national or international, supporting resources.

Within MELEES the availability of HELM Workbooks has enabled this resource to be made available for students:

i) global support   comprehensive access to additional learning materials readily available from an initial entry module.

ii) Module specific support - directed HELM workbooks within teaching modules

i) global support:

Figure 1: Initial display for Entry unit - MELEES

All students are provided with access to the Entry unit - MELEES - which provides general information on the Service Teaching provision, student learning resources and access to general support materials. Such materials can be used for supporting the mathematics students needs within their mathematics modules or additional topics they may meet within their Engineering courses. A further important aspect is to provide students with refresher materials to help in the transition from School/College to University. Figure 1 shows the top level access which is configured within an icon-based hierarchy.

Figure 2: Arrangement of HELM Workbook access in MELEES

Selecting the HELM icon provides students with a structured framework of Workbooks arranged in topic groupings that correspond to the 'Year' most relevant to the Engineering Mathematics modules that students will generally take or meet within their Engineering course. The selection starts with 'Foundation Topics' which corresponds to mathematical concepts to which intake students are typically expected to have competency. Topics selected in 'First Year' and 'Second Year' are based on the mathematics content of (compulsory) mathematics modules taken by students and those in 'Third/Fourth Year' are the more specialised topics associated with advanced/optional modules. Prominent on the display is access to the HELM Student Guide to provide students with information on the HELM project context and best-use guidance on using HELM materials. Experience in the development of MELEES indicates that many students prefer to download materials onto their own PCs for future use, which is particularly useful for students with visual impairment where they can use specialist viewing software or screen settings. For quick reference an icon is provided that gives ready access to any of the HELM workbook sections as illustrated in Figure 3.

Figure 3: Structured access to .pdf download of HELM Workbook sections

ii) Module specific support

Within MELEES each taught module has an individual entry unit that is available only to the relevant registered students. The module unit provides access to module specific materials identified by the module leader and incorporates a range of student learning resources that typically includes lecture handouts, example sheets, past examination papers etc., and sourced internally.

Figure 4: Entry unit for module Probabilistic and Numerical Techniques for Engineers

An example is shown in Figure 4 for a second year module 'Probabilistic and Numerical Techniques for Engineers' (HGBMPN) which covers both the basic theory of numerical solution of ODEs, probability distributions and an introduction to statistical tests.

Module staff can direct students to additional (external) materials, and trialling has been ongoing with HELM Workbooks. Figure 5 identifies the HELM workbooks relevant to this module and these are readily accessed from the icon 'E-learning Materials'. In this instance the module HGBMPN has ten workbooks relevant to the module topics and these are integrated with additional materials in this case an interactive self-test on use of tables of the normal distribution.

Figure 5: HELM modules selected to support the module HGBMPN

The flexibility of the HELM Workbooks aids considerably in targeting detailed support to students, particularly in modules that covers a range of topics as is typical in Engineering Mathematics teaching.

3. Using HELM in Assessments

The use of Computer-Assisted Assessment is an increasing area of interest both in allowing students to self-test on important topics (formative assessments) and also the administrative gains that may accrue from using CAA for summative assessments. A further crucial element in formative assessment is not only providing students with an indication of their strengths but also to identify their weaknesses and provide directed access to support.

At Nottingham, the non-availability of QMP restricted adoption of the HELM CAA question database. Computer-based self-assessment is a typical interactive feature within MELEES. An example is a Diagnostic Test which First-Year Engineering students are encouraged or required to take within their first week at University to provide teaching staff with an indication of cohort proficiency on Foundation topics. The test is of a multiple-choice format and questions are selected from a data-bank from five key topic areas. Feedback to students, such as shown in Figure 6, indicates an individual profile of competencies and importantly suggested sources for study materials to help

refresh any areas of weakness. The effectiveness of the test has been enhanced in that students can now be directed to HELM materials.

Figure 6: Typical output from Diagnostic Test directing students to support materials

4. Conclusions

HELM was a major curriculum development undertaken by the consortium of five English Universities. The follow-up transferability stage has enabled the range of resources to be evaluated and exploited at a range other Universities. At Nottingham University we have embedded the extensive range of high-quality Workbooks within our existing web-based support giving Engineering (and Science) students additional choices in their learning preferences. Further, the ready availability of the resources has enabled teaching staff to incorporate the resource within their own teaching. Recently Nottingham University has decided to adopt QMP as a web-based CAA-tool and this will enable Mathematical Sciences to evaluate further to use of the HELM databank of approaching 5000 questions to support student learning.

5. Reference

Hibberd S., Litton C.D. and Chambers C., (2006). MELEES - Reflective overview on use of a VLE in supporting student learning, Proceedings HELM Conference, Loughborough University, HE Academy Engineering.

6. The use of HELM at Oxford Brookes University

Contributor: Dr Robert Beale

Introduction

I transferred from the School of the Built Environment, Department of Civil Engineering and Construction Management to the School of Technology, Department of Mechanical Engineering in July 2002. For over thirty years the Civil Engineering Mathematics classes ranged between 20-50 students. The number of students in the Automotive, Mechanical and Electronic First Year module in 2002 was approximately 110 students. In 2002-3 this module was taught using lecturer prepared handouts and contained four class tests under examination conditions as well as a three hour end-of-module examination. The first year module is worth 30 CATS credits. The following year the module was taught in a similar manner with the addition that to each topic, as well as the lecturer prepared material, HELM workbooks were issued to each student. In 2004-5 all the workbooks were made available on-line via WebCT and the HELM CAA question bank used to replace four class tests with on-line quizzes. Finally in 2005-6 the online quizzes were amended to reflect changes in syllabus delivery and to add questions on topics not within the HELM CAA question bank. All lecturer material, HELM workbooks and solutions to lecturer prepared exercises and the last three examination papers were placed online. In response to requests from students, from 2004 the HELM workbooks were made online to second year students, directly accessible from their module home page. The second year module is worth 15 CATS credits.

First year students were always given printed copies of workbooks together with on-line access, second year students only had access on-line. In addition, following a request from the University Upgrade Service (a University-wide mathematics and English help facility centred in the Library each lunchtime during term time) and the Mathematics Department, in 2004 the HELM workbooks were made available online to all students within the university who could, in WebCT parlance, self-register for the HELM resources web-pages. Looking at last years registrations about 20 non-engineering students made use of this facility.

Workbooks

The workbooks are between 30-70 pages long and if distributed in paper form are best printed by the central print room. Difficulties were experienced initially because the print room required explicit copyright consent. In transferring the workbooks to other education establishments a statement on the printing and copyright conditions should always be included with the media.

When printing the workbooks in the academic year 2005-6 some earlier workbooks were included. This occurred because in previous years they had been printed but not used. These workbooks included solutions printed upside down. In addition all students had some workbooks with the questions printed with the solution the normal way up. It is interesting to note that strong feedback from students, including dyslexic students, found that when using the workbooks they found it easier to have the solutions printed up-side down rather than the normal way as they found that they could easily accidentally expose the solution and therefore lose the benefit of the question. [N.B. versions both ways round are now provided in electronic form.]

Neither students nor staff encountered any problems in distributing/accessing the pdf versions. In WebCT the University web manager simply created the equivalent of a module which was available to all staff and students to self register. Any module leader was given rights to release the files to all the students on any particular module. However, some students found difficulty in producing printed versions of the pdf files if they wanted to work off-line.

CAA files

In 2003-4 a decision was made to use the HELM CAA files. Loughborough provided zipped files of some of their databanks covering first year differential and integral calculus, trigonometry, complex numbers, exponential and logarithm functions, matrices and vector algebra. Each zipped file contained an xml form of the question databank and the jpg files containing the question image and the solution image. By a roundabout procedure the files were unzipped, imported into Respondus and then exported into WebCT. Each question of the database then had to be individually inspected and sets of answers included. The details are given in Appendix 1.

Oxford Brookes University was given transferability money to translate all the latest questions into WebCT which involved mounting a version of QM Perception onto a server at the University. The major difficulty encountered was due to the University having Unix servers. The Technician had to mount a Microsoft server before QuestionMark could be mounted. The University is currently developing a conversion procedure and will supply the finalised version to the HELM repository (Mathematics Education Centre, Loughborough University) when complete.

Many of the questions in the data set ask for two decimal digit accuracy numerical answers. It was discovered that if a question specified the input of a two decimal digit result and if the number in the supplied answer set was an integer or only had one decimal digit then WebCT/Respondus was unable to correctly import the answer. If the student inputted the correct two digit answer he/she was marked wrong! In every case manual editing of the question database had to be made to change the integer or one digit answer into a two digit answer and also to add alternative responses of the one digit or integer result with reduced marks.

Secondly when using a given dataset, depending upon the teaching order of the lecturer, the resulting set would need to be clearly checked to see that the students were correctly prepared and able to answer all the questions in the set. For example, the last section in the workbook for the exponential and logarithm functions also includes the hyperbolic functions. At Oxford Brookes University, exponential and logarithm functions are taught at the beginning of the course, followed by trigonometric functions and much later in the course, after differentiation principles and applications and simple integration techniques have been covered but before advanced integration techniques are taught, hyperbolic functions are introduced. This means that the databanks for exponential and logarithm, differentiation and simple integration have to be edited to remove the hyperbolic functions in early exercises. They obviously can be used in later on-line quizzes. This requires the lecturer to carefully check every question type in a database.

One of the major limitations of the databases is that most of the questions ask for numerical answers only. There are relatively few multiple choice questions asking for analytical answers, such as partial fractions or symbolic integration/differentiation. If these are required then the authors must prepare their own questions to cover these topics.

Feedback

Use of quizzes

Currently, preparing WebCT quizzes using the CAA databank has not saved any time over setting and marking traditional class tests. However, the major advantages that have occurred have been in meeting flexibility and disability requirements. The procedure that has been adopted in setting class tests is similar to that used in Loughborough. A sample test is available for four days which can be taken as many times as the student wants with an extended time limit on the test. For a period of 36 hours the test is then made available to all students. Each test requires the student to answer between 7-10 questions, each question being drawn from a bank of between 5 and 15 questions testing similar material. Students without disabilities have one hour to complete the test, those students with disability have 1.25 hours. The advantages with regard to flexibility are that the test can be taken anywhere within the university or off-campus and the test does not require staff supervision or special rooms being made available for those students entitled to special facilities. The tests are not monitored and hence I do not intend to use them for my second year module where the results can count towards classification. Currently students are only allowed one attempt at the test. I am going to allow multiple attempts next year in order to encourage the students to improve their marks. This procedure of allowing multiple attempts at assessments and recording the best mark has been used successfully in other disciplines within the university.

Feedback from students has shown that those students who enjoy working with computers like the tests, the others would rather have traditional tests. Secondly they have complained that a purely numerical result does not give any method marks and so they feel they have often done better than the test results imply and would obtain higher marks in a traditional test.

Use of workbooks

Oral feedback from students has shown that they value the HELM workbooks as giving additional opportunities to practise material within the mathematics course. Frequently during problem classes students can be observed working through the lecturer produced material and/or working through a HELM workbook. Students particularly like those workbooks which cover material which I had to omit from the lecture course as the students were assumed to have seen the material in previous courses. An example of such material is the introductory workbook on differentiation as a limit. I have to assume that students have met the calculus before. In practice there are a small number of students who, although they have taken National Diploma Mathematics courses for two years, appear to have not encountered the calculus, although it is within the standard syllabi. In addition they have valued being able to access the workbooks electronically.

Changes to teaching strategies during HELM introduction

It is impossible to isolate the effects of the introduction of the HELM project materials into the teaching of engineering mathematics at Oxford Brookes University due to the following additional changes which have taken place simultaneously:

- Between 2002-5 the mathematics classes were delivered by two lecturers, one concentrating on the calculus parts of the course and the other the remaining parts of the syllabus. In 2005-6 only one lecturer was used.

- Two research students were also employed to assist the lecturers when the students were split into problem classes containing approximately 24 students to a class. In 2002-5 each research student assisted a single lecturer. In 2005 the lecturer had to go between problem classes during the sessions.

- The university changed from a three term structure up to 2003-4 to a two semester structure from 2004-5 onwards. The introduction of semesterisation was particularly unpopular with engineering students as they felt it reduced the amount of study time.

- As part of workload planning changes, contact and problem class hours underwent significant reductions.

The results are summarised in this table:

academic year	student group	number of students	number of passes	lecturer contact hours	Problem class contact	class tests	final examination	HELM usage	module mean mark
2002-3	year 1	104	72	81	54	4 paper	3 hours	no	54
	year 2	80	55	54	36	2 paper	2 hours	no	49
2003-4	year 1	83	59	81	54	4 paper	3 hours	yes	43
	year 2	83	58	54	36	2 paper	2 hours	no	47
2004-5	year 1	61	41	66	44	4 CAA	2 hours	yes	45
	year 2	66	35	44	44	2 paper	2 hours	yes	47
2005-6	year 1	104	54	60	40	4 CAA	2 hours	yes	45
	year 2	45	34	40	20	2 paper	2 hours	yes	51

Note that the 2005-6 results are before examination committees and resits.

In addition to these changes to teaching of the engineering mathematics modules the university introduced in 2004 an 'Upgrade' service, centred in the library each lunch-time with additional support for mathematics which weaker students were encouraged to use.

Participation in lectures and problem classes remained the same over the period except that those in 2005-6 first year had much lower attendances across all modules. This behaviour is being investigated. Throughout the four year period students have made a reducing number of visits to staff offices outside timetabled slots. This may be due to more material being made available through WebCT during this time.

Dyslexic students who have always had additional support supplied by the university have stated that they like the workbooks and find them useful. However, they have stated that they prefer the earlier (upside-down) format of workbook solutions.

The class tests undertaken through the CAA databank have proved more popular with many students than the traditional pencil and paper test taken by the second years. They have taken significantly more time to prepare than paper tests but as I now have built up a reasonable number of questions I anticipate a reduced amount of time in the future. The students like the ability to take the test at a time of their own choosing and as a result of their comments I have slightly changed the

availability to ensure that they can take the test at a more convenient time on Saturdays. Their adverse comments are related to the difficulty of not obtaining method marks and the mark allocation for slightly incorrect answers. I will need to further investigate the solutions to produce improvements to marks for slightly wrong answers such as ignoring negative signs.

The amount of time taken to support the HELM workbooks is not different to that taken to support standard lecturer material. Students have asked for assistance in using the workbooks at the same rate as using traditional materials. Students have expressed a strong liking to being given the choice between working through a HELM workbook and working through lecturer supplied material. The amount of assistance required has not changed as for many students the HELM workbooks are considered to be supplemental material; others use it as their main material.

Critical issues

All lecturers who have been involved in delivering the engineering mathematics courses have found no difficulty in using the materials and have readily given out the handbooks when required. The complex number workbook was used by one lecturer in preference to writing new notes but with additions relating to the use of complex numbers in electronic circuits.

The costs of printing workbooks are high and I have faced criticisms from within the department about the costs of the module. The module has the highest paper cost per student within the department. In order to keep the costs down I have taken to printing workbooks close to the time when they are being required so that the number of copies can be kept in line with module attendance. Putting the workbooks online has meant that students do not always bother to collect a workbook but simply work from their own or university computers, hence reducing the overall demand. There have been technical difficulties in printing online copies of workbooks by some students on some computers.

The major problems with the workbooks come from the sequencing adopted. Firstly, although exponential and logarithm are correctly introduced early in the sequence of workbooks, it seems odd that hyperbolic functions are included in this workbook as many students needing exponential and logarithm functions are still not fully conversant with trigonometrical formulae and have not covered complex numbers so that the link between trigonometric functions and hyperbolic functions cannot be demonstrated. These students would find the early use of hyperbolic functions confusing. Secondly, partial fractions are introduced outside of their use in integration which would mean that engineering students, who need motivation, would not see one of their common applications. In general, individual workbooks are well written and their contents graded. However, the author of the differentiation workbook assumed that he was revising the product and quotient rules and the examples in the exercises rapidly get very complicated. In addition the exercises are written assuming that every lecturer is following the full sequence of workbooks. This leads to, for example, exercises in differentiation using hyperbolic functions before an alternative sequence would cover it. I have also found that the use of inverse functions is scattered throughout the text and many students do not fully appreciate them and still confuse the inverse function with reciprocals. In some workbooks there is an excess of material for my class. However, I do not feel that this is a problem as other courses will include different sub-topics. This is not different to any textbook. The students are notified that a particular sub-topic is not going to be examined.

It is a long time since I found an error in a workbook. The authors and editors are to be commended for their attention to detail and accuracy. Most workbooks develop a topic in a sensible manner with

the major exception mentioned above regarding hyperbolic functions.

Students have fully engaged in workbooks and I have found second year students printing off and using relevant workbooks with very little prompting.

The major limitations of the CAA questions have been described above with regard to handling two digit answers when the supplied QuestionMark answer is either a single digit or an integer, assigning marks to partially correct answers and the difficulty of translating the questions into WebCT. However, a justifiable criticism of the questions levelled by students is that the majority of questions require a numerical answer. I have had to develop questions involving symbolic integration and partial fractions. In addition, the questions on trigonometry supplied only covered wave theory. I therefore produced questions on other aspects of trigonometry. These additional questions are available to anyone requiring them. Oxford Brookes University will, over Summer 2006, develop and implement procedures to translate the existing QuestionMark databases into WebCT. The resulting files will be distributed via the HELM administrator. An appendix to this report shows the method of converting older QuestionMark files.

Conclusions

Using the HELM workbooks has enhanced the student experience. Student find them easy to use.

A significant effort is required to adapt the CAA files for use on non-QuestionMark systems but the CAA files can be translated.

Appendix:
Detailed instructions on converting old QTI questions into WebCT

The following procedure was adopted at Oxford Brookes University to convert QTI questions sent from the HELM Project in 2004 into databases for WebCT.

1. Unzip the files into a temporary directory.

   e.g.   fn_10_02_02_argand_diag

2. Within the temporary directory use Notepad or equivalent word processor to edit the .XML file as follows:

   - find   % SERVER.GRAPHICS %
   - replace with   /_COURSEID_/*local_folder*/
   - save

   In the above example *local_folder* could be *argand_diag*

   Note the trailing '/' at the end of the replace and also that '_COURSEID_' must be WebCT directory.

3. Enter RESPONDUS and perform the following steps:

   - Change "personality" to IMS QTI. This is found under the START tab.
   - Import questions.
   - Whilst in this sub-menu Browse to the temporary directory containing the XML file and browse to the temporary directory containing the images folder.
   - Create a new document names as *local_folder*.
   - The 'stem' is also *local_folder*.
   - Finish
   - Under the START tab change "personality" to WebCT 3x4x.
   - Replace current file.
   - Under the PREVIEW AND PUBLISH tab - publish to WebCT.

4. Copy the .fb or .sb files from the unzipped temporary directory to the WebCT directory containing the question bank using either WebDAV or my-Network_places.

Note when composing WebCT tests that you must carefully look at the answers that have been imported. I noticed that a question which asked for a two digit answer with an answer which was either an integer or 1 digit was converted to a one digit answer. WebCT quiz then marked the two digit or an integer response as wrong. Only a one digit response was given a correct mark. I am currently trying to get from WebCT a procedure to enable a range answer to be accepted. You need to edit the allowed answers to give correct marks for all possibilities that you want to accept.

7. The use of HELM at the University of Portsmouth

Contributors: Lynn Pevy, Fiona Message, Dr Michael McCabe

Introduction

The University of Portsmouth first used some of the HELM workbooks with approximately seventy second year Electronic and Computer Engineering students in the 2004-2005 academic year. These workbooks were well received by the student cohort, and seen by the lecturer concerned to be a valuable resource which would be also benefit other groups of students. Portsmouth was keen to become one of the HELM Transferability Partners in the hope of encouraging greater awareness and use of the materials, and appreciated the opportunity for greater involvement in the 2005-6 academic year.

Use of HELM Workbooks

The only HELM materials extensively used at Portsmouth have been the workbooks, and these have primarily been issued as paper-based workbooks directly to the intended student users. They have been distributed in this form to first and second year Engineering students and to second and final year Mathematics students. The manner in which the workbooks have been used has been different for the various student groups. At one extreme the materials have been provided for independent study as one of a number of resources, and at the other extreme the materials have been deliberately allowed to exert a controlling influence of the course delivery. All the lecturers currently using HELM materials are members of the Mathematics department. However, some individuals in other departments have expressed particular interest in using the materials themselves.

Summary of HELM Workbook Usage in the 2005-6 Academic Year			
Student Department or University Facility	**Unit of study**	**Year**	**Workbooks used**
Electronic and Computer Engineering	Engineering Analysis	2	9 (Matrices) 20 (Laplace Transforms) 22 (Eigenvalues) 23 (Fourier Series) 24 (Fourier Transforms) 21 (z transforms)
Mechanical Engineering	Mathematics	1	6 (Logarithms) 10 (Complex Numbers) 11 (Differentiation) 13 (Integration) 14 (Applications of Integration)
Electronic and Computer Engineering	Engineering Analysis	1	13 (Integration)
Mathematics	Calculus of Several Variables	2	28 (Differential Vector Calculus) 29 (Integral Vector Calculus)
Mathematics	Complex Analysis	3	10 (Complex Numbers) 26 (Complex Variables)
Maths Café	Various	Various	Miscellaneous extracts

Access to the pdf versions of the workbooks has been provided for all lecturing staff. For students access has been provided, when requested, to pdf versions of those workbooks which have not already been distributed to students in paper-based format. The two Engineering departments mentioned in the table above also run some high level courses that include an element of distance leaning over the summer months. It is anticipated that the HELM material will be made available in pdf form to the students on these courses over the coming months (i.e. Summer/Autumn 2006).

The HELM workbooks were one of a number of resources provided for independent study for a second year mathematics unit on the Calculus of Several Variables. A diagram of the integration of HELM with other resources is shown in *Appendix A*. In this unit the students are required to submit learning diaries containing a record of their work, and their answers to HELM questions were expected to form a part of this. The lecturer was concerned about the fact that the answers in the workbooks are in close proximity to the questions. It would have been helpful for the lecturer to have had access to a copy of the workbook with white spaces instead of answers. In this instance the answers to all the exercises were blanked out by hand before the workbooks were distributed to the students. The answers were later made available to the students via WebCT. The HELM materials were well received by the students and extracts from their learning diaries are included in *Appendix B*. [Note: It has been arranged to provide, from Summer 2006, all workbooks in three electronic formats: Task solutions the right way up; Task solutions inverted; Task solutions blanked out.]

The other instance where HELM materials were used entirely for independent study was in a final year Mathematics unit in Complex Analysis. In this case workbooks 10 and 26 were handed out as a starter pack for the unit. In week one of the course the students were advised to work through Workbook 10 in order to revise work that they had met some time previously. Workbook 26 was also handed out at the same time and the students were told that this contained some of the basic material for the course, and they would find it useful to complete the exercises in HELM before continuing to more advanced exercises to be distributed on tutorial sheets throughout the unit. The lecture course then continued entirely independently with only the occasional reference to the relevant section in workbook 26. At final year level this proved to be an entirely satisfactory use of the materials. When the lecturer enquired whether anyone had any questions relating to the content of the HELM material the response was 'Why should we? The answers are all in the pack'. When asked about the materials the only negative comment was that there were some typographical errors.

Portsmouth provides a facility through which students of all levels and from all parts of the University can access one-to-one mathematics support. This facility is called the Maths Café, and a complete set of the HELM workbooks is kept as a resource in the Maths Café baseroom. (Maths Café operates in two ways: staff take a laptop into a student café area and students just come along for help; students visit the Maths Café baseroom at allotted times.) Almost all Mathematics Department lecturing staff (and also some research students) have timetabled hours in the Maths Café, and the expectation is that the staff on duty will endeavour not only to answer the immediate maths problem that might have brought the student to the Maths Café, but will also try to identify other useful resources which the student might also find helpful. The HELM workbooks now constitute such a resource. The Maths Café keeps stocks of those workbooks known to be in current use by various student groups, but does not keep stocks of additional workbooks. If a section of a workbook is identified as a potentially useful resource, then the relevant material is photocopied, and the student is sent an e-mail to say that it is ready for collection from the Maths Café baseroom. During the year all Maths staff had opportunities to hear about and view the Math Café resources at a staff development lunchtime session and again at the Christmas Maths awayday. The potential for using

HELM materials was emphasised by the Maths Café team at both these events. The HELM materials were also highlighted at two Maths and Engineering Teaching Forums held in 2005-6. There was considerable interest in the materials from both departments. In particular the additional engineering examples workbook (48) was seen as valuable by the Mechanical Engineering staff.

The Maths Café has had difficulty in finding good resources for statistics. The advent of the Statistics workbooks was therefore greatly welcomed by Maths Café staff.

Workbooks 9, 20, 23, 24 and a large part of workbook 22 were printed and distributed to year two Engineering students in semester one. This amounted to a considerable volume of paper, and of necessity resulted in students following this unit having an even more extensive and comprehensive set of personal notes than the lecturer might previously have thought necessary or desirable. Unfortunately, with the exception of workbook 22 (Eigenvalues & Eigenvectors) the excess material not needed for the course was not completely contained in a section that could easily be omitted. This might have caused students using the workbooks independently to cover too much material. To overcome this problem the students were provided with a study schedule that clearly indicated which sections were essential, and which sections could be treated as background reading and were optional. The schedule also identified those questions which the students should attempt from HELM, and when they would be provided with additional questions. This was a satisfactory compromise in terms of student learning. However, it should be noted that students were being provided with some notes that were likely to be viewed as redundant by many students, and the photocopying of such material therefore constituted an unnecessary cost.

The second year Electronic and Computer Engineering courses traditionally attract a high proportion of direct entry international students. These students place particular value on printed notes, as English is the second language for many of them, and making notes while following a lecture is even more of a challenge to them than it is for those for whom English is their native language. The decision taken by the lecturer was to try to refer to the workbook examples as much as possible during the lectures in order not to confuse students and to avoid adding significantly to the quantity of written material collected by the students. Occasionally the lecturer felt the notation used or the presentation in the HELM workbook was not as good as the way the material had previously been presented. In order to avoid confusion some compromises had to be made in the interests of conformity with HELM format. A particular case of a conflict of interest was in respect of the formulae provided. The Portsmouth standard Maths and Stats formulae book was to be made available to students in assessments as well as additional formulae sheets including Fourier Series formulae and tables of Laplace and Fourier Transforms which followed the HELM notation. This resulted in duplication of information provided, and students had to be advised to use whichever source they found most useful. Our intention is to produce a new formula book specifically designed for Engineering students that contains formulae and standard tabulated results in exactly the same format as that used by HELM.

On first year Engineering Mathematics courses taught by two different lecturers, the lecture course was not altered greatly to suit the HELM material, but the students were directed to selected workbooks throughout the course. The second part of the report contains a full record of one lecturers personal experience of using the HELM material in this manner.

There has been no antagonistic resistance to the introduction of HELM with the exception of the statistics workbooks, the causes of which appear to be based in notation and are being investigated and, it is hoped, addressed. To date, some staff have been more cautious than others in the way in

which they have used the materials in lectures, but even non-users have recognised that the workbooks are a resource to which engineering students (and others) can be directed if they need to catch up on missed work, or can review work that is poorly understood. The use of the HELM material is not expected to remain the preserve of the informed, and use of the material is expected to continue to grow. We can be confident about this because, in the Maths Café and its resource-update staff development sessions, Portsmouth has an excellent mechanism for keeping the material permanently in the sight of all members of the department and reminding them of its potential.

Use of Computer Aided Assessment

Portsmouth Mathematics department has a fairly long tradition of using Questionmark Perception tests for both formative and summative assessment in various units for Mathematics and Engineering students. The Portsmouth sample (practice) tests are made available for students to access at any time, but the summative assessment are conducted under examination conditions. The examination conditions are imposed because the summative assessments are not taken at the same time by all students, and also because the assessments can be retaken a limited number of times by any student wishing to improve their mark. This system was introduced to encourage students to engage with learning the essential material early in the semester. However our experiences have demonstrated the necessity for CAA summative tests to be run with diligent invigilation. The production of Questionmark Perception test question databanks has already proved very time-consuming, and there is always student demand for additional practice tests. This meant that at the start of the 2005-6 academic year we were very keen to use the HELM resources to supplement our existing resources, particularly with respect to formative assessment. Unfortunately this did not happen as planned. A local delay in the resources being made available for lecturers to review resulted in the CAA material not being used by any HELM users in 2005-6, Although some lecturers looked at the resources, none found themselves able to integrate the resources into their delivery schedules after the term had started. In essence this was the one area where in 2005-6 we did not use the materials as originally intended, However the resources are now accessible to potential users of HELM materials, so for the 2006-7 academic year we expect to be integrating the CAA tests into the courses for some groups of students.

There is considerable interest in the potential for the use of MapleTA at Portsmouth. The advantage of MapleTA is that a once a question has been written it can be reused extensively as parameters can be automatically changed to produce a similar question assessing the same essential skills and knowledge. If the HELM questions were in this form, then we would be able to use the same questions in both formative and summative assessments. One lecturer whose students have made considerable use of HELM workbooks over the last semester has now undertaken the task of writing some of the HELM questions in MapleTA. This is a major task, but one which, in the long run, could result in a considerable reduction of the staff time which must be allocated to this form of assessment.

Summary of feedback from students and learning support staff

Given our relatively limited use of the materials, and the natural variation in student cohorts between one year and the next we cannot attempt to produce quantitative data to demonstrate an improvement in student performance. However, student feedback clearly demonstrates that the students consider that the materials have contributed greatly to their understanding, and have valued the impact that this has had on their learning experience. Maths Café staff have also welcomed the resources.

Feedback from lecturers

The materials cannot possibly fit all purposes exactly, but we have been pleased to note that materials designed for Engineering students have been so well received by Mathematics undergraduates.

In some case there is too much material and it would be good to be able to delete sections, and sometimes the material has to be supplemented by material from other sources.

Typographical errors in a few workbooks irritated both students and lecturer. [Note: where known these are being rectified for the Summer 2006 re-issue.]

It would be helpful if the workbooks were available in a form that would allow for some local variations to be made (additional questions, deletion of unnecessary sections, changes in statistics notation, correction of odd typographical errors). [Note: such requests will be considered although it is HELMs policy not to encourage changes to the materials to maintain quality control.]

It would also be helpful if there was an alternative set of books in which the full solutions were not given in close proximity to the questions, and the spaces currently occupied by the solutions were replaced by blank spaces. If it were possible to produce such a version, then additional separate books of full solutions would also be helpful. [Note: It has been arranged to provide, from Summer 2006, all workbooks in three electronic formats: Task solutions the right way up; Task solutions inverted; Task solutions blanked out.]

A Portsmouth Lecturer's Perspective on using HELM

I have introduced HELM material for the first time this year to first year Mechanical Engineering students. The students are taught in 2 groups about 25 on the accredited course, and 20 on the lower level non-accredited Mechanical and Manufacturing course. The syllabus for the first year maths is the same for each cohort (approximately A-level core plus a few extras) but the tests and exams have questions set that reflect the difference between the 2 groups. Since some of the smaller group have no post-GCSE mathematics experience and some of the other group have further maths A-level, there is a wide range of ability.

I have never used a set text book before, or handed out printed notes, as I think, on the whole, that students concentrate more if they are copying examples as we go through them, rather than just looking at a piece of paper. However, I liked the structure and layout of the HELM workbooks and felt that it would make a change of delivery and provide some variety during the year if I made use of some of them for suitable topics.

The following summarises the advantages (or otherwise) of the workbooks as I see them my student feedback follows later.

Plus points

- The students liked having a whole topic 'in front' of them so they could see a clear beginning and end (!).

- I liked having background information on a topic so that I didnt need to waste class time reading it out.

- Students that miss a class didn't lose out on the notes.

- The books provided a good revision source for the exam though most students seemed to only look at past papers, and not revise particular topics!

Minus points

- I dont like being restricted by someone elses idea of exactly how the topic should be taught.

- There is a danger that students may collect the workbook and then not attend classes so I was careful not to let them know how long we would spend on any particular topic!

- There were not enough exercises on any particular topic so I still had to give out photocopied exercises.

- It would be expensive for photocopying/printing to use the books exclusively.

Student Feedback to a questionnaire

Of the 25 responses the feedback can be summarised as follows:

'Positive features of HELM ?'

- Clear explanations - 8 students

- Portable 3 students (though Im sure more would have agreed with this)

- Easy to use 6 students

- Clearly laid out 11 students

- A lot of, and useful, examples 13 students

- Solutions included 10 students

- Other points included:
    Need to take fewer notes
    Gradual increase of difficulty
    Formulas always given.

'Ways to improve HELM ?'

- More exercises 11 students

- Provision of related tutorial sheets 8 students

- Solutions should not be with exercises 5 students

- Layout was mentioned with suggestions to use colour (!), spread things out more, use bold type, use laymans terms, more space for exercises.

- Only one student wanted more theory!

'How do you best learn Mathematics ?'

- Attending lectures 3 students (I'm sure more would have agreed with this I hope!)

- Being shown an example and then practising 16 students (phrased in various ways)

- Other answers:

  By osmosis ! (If only !)

  Working with others

  Practising at home (only 1 student)

  With a lot of help

  Using handouts

  1 to 1 tuition.

Of course it's no surprise that students say they learn best by DOING maths. It is a bit surprising that none of them mentioned a textbook or other external resources.

Overall the HELM books have been well received. The fact that the students were able to comment on the explanations given means that they have looked beyond the examples, which is positive.

Addressing the main complaint of the students that the books are short on exercises/ tutorial sheets - should be quite straightforward. Since the books are downloaded as and when required there is no difficulty in attaching extra sheets (or removing some of the areas I dont want to cover).

<u>Future Use</u>

I will certainly use the books again, albeit in a limited capacity. They are well constructed and provide variety. For certain topics in particular (for example Complex numbers, Differentiation, Integration, Differential Equations) it will be useful for the students to have the foundations in front of them so I can concentrate on the application of the process without having to spent valuable lecture time on theory that they will never have to 're-create' and of which they only need a basic understanding. I very much believe that I am teaching students to use Maths as a tool I am not attempting to make mathematicians just good method followers!

As far as other HELM resources go I have yet to be convinced of the benefits of on-line assessment as I prefer to see how students have arrived at their answers and with smallish groups I have the luxury of being able to do so. The Questionmark Perception tests would be ideal for individual formative testing, but, unfortunately, my students rarely do any work that is not marked, so I cant see them taking advantage of this resource.

Overall the HELM experience has been a positive one, and I look forward to receiving the latest (and last) edition.

Appendix A

MTH204 STUDY RESOURCES

MAPLE 10

Check written solutions
Visualise written solutions
Solve harder problems
Explore tools and learning aids
(learning diary)

WebCT
Access to resources including
Mathwise and Maple samplers
Discussion board
(learning diary)

Mathwise

Computer based learning
Interactive presentation
Practice questions
(learning diary)

Computer marked practice test

Prepare for computer marked
assessment

HELM Workbooks and Study Pack

Basic study resource

Past Test Paper

Prepare for written
assessment

Anton Textbook

Detailed explanations
Many questions for learning diary

Short answer questions
Answers without working
provided
(learning diary)

Multi-Choice Questions

Answers without working
provided
(learning diary)

Appendix B

Section 1 - Comments from Maths students using HELM for independent learning together with other resources

1. I used the recommended text quite a lot, but HELM provided better working out and simplified examples.

2. I did make full use of HELM as I attempted nearly all the tasks. Towards the end of the unit I realised how important the examples and the explanations in HELM were, this is because I found the last weeks work harder. I found the HELM books supplied good examples and information throughout.

3. I did not like (names another resource) but I really enjoyed the HELM workbooks . . .

   My favourite part of the course was completing the HELM workbooks as I think they were very well presented and easy to use. Although I struggled with some of the questions and found that a number of the answers were incorrect, I did find them easy to use and I think that they are a good way to learn the material.

4. The HELM booklet was a reasonable source of information but was not very in-depth, which was off-putting for me to use it again.

5. I made extensive use of the HELM and recommended text. For each week I began my learning by completing the HELM exercises, followed by use of the recommended text.

6. HELM being my best friend throughout, as it was really easy to follow and gave excellent examples in order to complete the questions.

7. The HELM workbook was good as it had a variety of different questions and most of the material was explained clearly and I understood what was happening. There did seem to be a few mistakes in the workbook.

8. I mainly used the HELM material and the recommended text. This was because the HELM material is great to get you started on what you need to know and then the recommended text follows on from HELM and gives you even more questions and more in depth descriptions of the material being learnt.

9. Overall HELM was useful, but not detailed enough.

10. I only really used the recommended textbook, as HELM was very complicated to understand. The worked examples were not as straightforward as the recommended textbook.

11. HELM workbooks my studies were based around these, they were straightforward and easy to follow and had lots of worked examples before challenging you to do some.

12. I used mainly HELM, although I would use the recommended text every now and then to clarify things. I tried using Maple and Mathwise in the early stages of the term, however I found them to be confusing and more hassle so I just used HELM.

13. The HELM workbook is exceptional! I cannot emphasize enough how helpful it has been. It is a brilliant piece of work.

14. The HELM workbook was the main learning tool that I used and most of the time it was fantastic but some weeks I did not find it as helpful. This was due to the huge jump in difficultly between the examples I was shown and the questions set for me to do. In the weeks where HELM was not as good I turned to the recommended text.

15. I found the HELM workbooks well-paced and accessible so concentrated my studying there and found this sufficient to gain full understanding of nearly everything involved. On a couple of topics I referred to the recommended textbook for an alternative explanation just to reinforce what I learned from HELM, and found this a well-balanced and effective contrast.

16. I struggled with some of the areas in this unit and have had to work hard in order to understand concepts and methods. Overall the HELM was useful, despite some unclear patches. In general I only used HELM to aid my learning.

17. My preferred learning resources were HELM and the recommended textbook. I found together they provided a comprehensive and solid basis for learning.

Section 2 General Comments from Engineering students using the HELM materials as the main resource for their unit

1. Are there any HELM workbooks relevant for my semester two Maths unit? I found the workbooks useful in semester one and would like to continue using them.

2. HELM is a good reference for Maths and analogue electronics. The workbooks provide electrical circuit examples, which is useful for my degree subject.

3. Quite different from other resources. It gives more examples and solutions to questions (although at times the solutions are not detailed enough).

4. From an Engineering perspective HELM fits very well in the environment as the notation is focused on the Engineering aspect rather than pure maths.

5. HELM is my first source of reference. I either use it alone or to supplement lecture material.

Summary of good features of HELM identified by students:

- Well structured. Clear progression of material.

- Good presentation clear, bold and easy to read.

- Good use of graphical examples.

- Learning outcomes and pre-requisites are clearly stated.

- Simple and straightforward to use. You can pick it up and make your way through it.

- Comprehensive coverage - it covers pretty much every aspect in detail.

- Good examples

- Inclusion of proofs is helpful

- Key Points help me locate the material I need.

Student suggestions on ways to improve HELM:

- More emphasis on important points.

- Add white spaces around critical material, particularly formulae. Some sections look packed in.

- More examples.

- Answers at the end, not just below the questions.

- Paragraphs are too long. It can be too much. Some explanations are too full, and only the Key Points should be included.

- More thorough fully-worked solutions to problems. Some steps which may seem obvious need to be included.

- More background in general. I would have benefited from more fundamental explanations.

8. The use of HELM at Salford University

Contributor: Dr Ian Drumm

Introduction

This report pertains to the delivery of mathematics to students doing acoustic related degree programmes in the department of Computing, Science and Engineering at the University of Salford. Salford has been carrying out teaching, research and consultancy in acoustics and audio for three decades offering a range of degree and postgraduate courses in acoustics, audio and video. The associated acoustics research centre gained a 6* in the last RAE.

The main degree courses offered BSc (Hons) Acoustics', BSc (Hons) Audio Technology' and 'BSc (Hons) Audio, Video and Broadcast Technology' typically have the high mathematical content associated with elements of classical physics, signal processing, electronics, computing and experimental methods/analysis.

HELM materials were adopted to directly assist in the delivery of a 20 credit undergraduate mathematics module at level 1. Though some of the more advanced materials were also offered to support acoustics modules with mathematical content at Level 2, 3 and MSc.

Issues with the past delivery of Mathematics

Over the last the decade there had been a steady increase failure rates in the level 1 mathematics module associated acoustics degree programmes. This trend could also be observed in the more physics orientated acoustics modules at level 2 and 3 where a strong mathematics pre-requisite is essential.

To some extent these problems can be related to the ever-increasing need to offer flexible recruitment paths to include students with a strong interest in acoustics and audio yet without a strong A-level grade in mathematics. Indeed some 40% of our students are now recruited without A-level mathematics but are instead recruited with A-levels that include a 'Numerate Science'. The 'Induction Week' test in Autumn 2005 based on elementary algebra, trigonometry and some very simple calculus yielded 42% getting below 40% in the test.

There was clearly a degree of introspection required as mathematics delivery was becoming less successful at meeting the needs of the students. It was decided to investigate key problems with mathematics pedagogy in the hope that different delivery, assessment and support strategies might mitigate for poor mathematical knowledge or acumen. A number of specifically tailored surveys were conducted; these included interviews with

1. students to establish needs, difficulties and expectations

2. module coordinators to gain a more focused picture of prerequisites

3. 41 employers to gain a clearer view of their expectations of student attainment.

It was also felt important to share experiences and resources with other institutions to better establish best practice.

Some key considerations became clear:

- Delivery should not sacrifice a range of content that would still be vital in subsequent modules.

- Delivery should be in a way that builds confidence and interest in weaker students.

- Delivery should provide challenges for stronger students and thus encourage their participation throughout.

- The mathematics content should be immediately relevant to the student with respect to aims and objectives of an acoustics degree programme and their chosen acoustics/audio related career path.

- Students should be encouraged to exploit wider support mechanisms and resources beyond lectures and tutorials.

- Students should be gently introduced to the concepts of student centred learning and ownership thereby to gain the learning skills that would be vital when more difficult mathematical content is presented later.

- Some peer support and mentoring should be facilitated.

- Students need to be encouraged to work consistently, and opportunities for formative feedback should be exploited.

- Changes should be manageable with limited staff resources and time.

Previously, 'Level 1' mathematics was presented as two 10 credit modules in semester 1 and 2. Assessment was a summative exam held at the end of each semester. Students were expected to take notes in classic 'chalk and talk' fashion and encouraged to engage with some maths problems during the two hour lecture period. Although some bright or highly motivated students would find this style an efficient way to guide their learning, many poorer students would devote their concentration to note-taking, fail to follow the chosen pace of the lecturer, have little knowledge of expectations and not devote enough of their own time to learning.

The New Module Structure

The change proposed a new 20 credit module structure. It was decided that HELM resources would play a central role hence the new module would make timetabling and resource provision for workbooks and computer aided learning / assessment.

Workbooks

Key to the delivery of the module would be the HELM workbooks. These were the main learning resource defining the teaching content. The exercises would also outline the likely assessment expectation. Students would be able to work through workbooks during lectures and in their own time.

Hard copies of the workbooks were given out in lectures to students during the progression of the module (typically one workbook per fortnight over the 24 week period). Level 1 workbook topics are listed below; which are essentially a combination A-level revision and elementary degree level mathematics. Other important workbooks dealing with the likes of matrix algebra, Fourier series, Fourier transforms Laplace transforms, z-transforms, probability, statistics etc. would be available to students / lecturers as support materials later.

Week	Topic	Workbook	Assessments
Semester 1			
1	Basic algebra	1	
2			
3	Functions	2	
4			
5	Equations, inequalities, partial fractions	3	
6			Phase Test A
7	Trigonometry, coordinate systems and series.	4	
8			
9			
10	Logarithms and exponentials	6	
11			
12			Phase Test B
Semester 2			
1	Complex numbers	10	
2			
3			
4	Differentiation	11	
5			
6			
7	Integration	12	Phase Test C
8			
9		13	
10			
11	Differential Equations	19	
12			Phase Test D

Lectures

There were two hours of lectures every week. Power point slides ($\sim$20 per lecture) were created would follow very closely the explanations and worked examples presented in the workbooks. This was important to give the students the feeling that the workbooks were central to their learning as opposed to yet more support material. Exercises provided an excellent way to break up the lectures into three 20 minute periods of talk punctuated with opportunities for student application, inquiry and recall. The author found it useful to provide, in addition to answers provided in workbooks, the school's own worked solutions given as handouts during lectures. Clearly many students wouldnt complete all of the exercises during lectures so external study and use of Mathscope (the Universitys maths support unit) was encouraged. Students doing badly in early phase tests were asked to have signed receipts to confirm their visits to Mathscope.

Assessment

Although HELM's computer aided assessment was not used summatively, the module's assessment strategy was re-structured to make use of the formative feedback potential of CAA (as will become clear). The module is assessed as follows:

- There were four Phase Tests (A, B, C, D), two in each semester. Each counted for 20% of the marks for the module.

- If a student passed all Phase Tests then he/she would not need to take the examination in May/June. Otherwise a formal exam would require students to complete exam question sections corresponding to topics assessed in those phase tests they had failed.

- In addition there was a Group Assignment, which counted for the remaining 20% of marks for the module.

  A student failing to attend a phase test and then the subsequent exam if required or failing to contribute towards the group project would suffer a large marking penalty to discourage students skipping components and playing the numbers game to clear 40%.

To avoid resource intensive marking, two postgraduate students were paid to mark phase tests based on clear assessment criteria with questions tailored to make quicker marking easy. Computer Tutorials In addition to the two hour lecture slot there was a one hour session in a Computer lab timetabled for a later part of each week. Here students were able to familiarise themselves with important tools (notably Matlab) to help understand the application of mathematics in acoustics. These sessions followed the workbook topics closely where computer exercises would try to marry the workbook topic of the week with acoustic application and Matlab (e.g. to draw graphs, synthesise sounds, etc). Computer tutorials were also an opportunity to try other HELM resources such as interactive lessons and computer aided assessment. Although CAA was not used during formal phase tests; online practice tests were constructed and a 'mock' phase test run for each week prior to a formal test. These proved very popular as a confidence building exercise. Given that a cohort would typically consist of 60 students, two separate computer rooms of 35 places had to be run simultaneously. We decided to take advantage of this constraint by streaming the students according to the results of the mathematics test in induction week. Students in the weaker group B tended to prefer the hour used for extra tuition with workbook exercises where as group A students preferred the creative application of Matlab. It should be noted that students were also introduced to the online videos at mathcentre.ac.uk and some students liked to access these briefly on a weekly basis.

It should be noted that use of computer labs tended to distract some weaker students from the learning outcomes of the module; when they got stuck with Matlab or simply refused to engage they used the computers for other purposes instead. Also, attendance in the Computer tutorials has been low with an average of less than 30% of the weaker students turning up. This suggests that although the HELM workbook material can be very effective, the computer laboratory it is perhaps not the best place to engage with it for some weaker students. In addition to the exercises provided in the HELM workbooks, it turned out to be useful to the weaker students to be given additional exercises to get more practice.

Reflections on the use of HELM resources

Students An online survey was conducted where students were shown twenty statements pertaining to Level 1 mathematics delivery in the department. We recorded 35 participants for one day. They were asked to select from five options

1, Strongly Disagree, Disagree 2, No Opinion 3, Agree 4, Strongly Agree 5

The following table shows the averaged student opinion, so for example a statement scoring 4.5 will have strong agreement from the students.

1	Mathematics is one of my strongest subjects	3.6
2	Mathematics is an interesting subject	3.9
3	I believe at degree level learning is the responsibility of the student	4.0
4	Topics presented during the level 1 module are too basic	3.0
5	Topics presented during the level 1 module do not sufficiently address the basics	2.3
6	I have or will find extra maths support (e.g. Mathscope) useful	3.6
7	I am better able to understand mathematics topics because of the HELM workbooks	3.4
8	The HELM workbooks are clear and informative	3.6
9	HELM workbooks arent sufficiently concise	2.7
10	HELM workbooks are too concise	2.4
11	HELM workbook exercises are too easy	2.4
12	HELM workbook exercises are too hard	2.6
13	Lecture sessions that closely following topics in the workbooks are appropriate	4.3
14	I have found HELMs computer aided learning resources useful	2.7
15	I have found HELM online practice tests useful	4.0
16	I would be happy with purely computer based assessment	2.9
17	I would like access to HELM workbooks covering more advanced topics at later stages of the degree programme	4.1
18	I think a number of assessments throughout the year more appropriate than a single exam	4.6
19	I find other online resources (e.g. videoed lectures) useful	3.3
20	I dedicate much of my own time to working through workbooks	2.4

From the high scores on items 3, 8, 13, 14, 15 and 17 we conclude that students respond positively to HELM workbooks as an aid to learning.

The statement '*I am better able to understand mathematics topics because of the HELM workbooks.*' was tempered by three of students who gave 1s. These particular students exhibited a similar response to other resources including Mathscope and rated their own mathematics ability with 5s. The modal average for this question was a very positive 4. The three outlier students did however give very positive responses to the high scoring statement '*I would like access to HELM workbooks covering more advanced topics at later stages of the degree programme.*' reflecting a near unanimous recognition of the potential value of the workbooks.

The statement '*Lecture sessions that closely following topics in the workbooks are appropriate.*' had a particularly positive response from students suggesting the practice of writing Powerpoint slides that mirrored the explanations and examples in the workbooks was appreciated by the students who

were clearly happy to see HELM play a central role in their learning. Although it is tempting for mathematics tutors to stick to previous lesson plans and slides and thus offer HELM workbooks as a supplement, the author would instead strongly recommend an overhaul of Mathematics delivery to make HELM the central component.

Interestingly, the students agreed strongly with the statement '*I believe at degree level learning is the responsibility of the student.*' yet disagreed with the statement '*I dedicate much of my own time to working through workbooks.*'. This suggests they buy into the expectation of student centred learning but admit they arent doing it! Only 15% of students gave 4s or 5s to the latter statement which was a surprise given the workbooks are ideal for revision and self learning.

It is also worth pointing out the statement with the highest agreement '*I think a number of assessments throughout the year more appropriate than a single exam.*' reflected a positive response to continuous assessment. Another statement '*I have found HELM online practice tests useful.*' also exhibited a positive response showing the value of HELM computer aided assessment as a mechanism for formative feedback. The students however had reservations for this to be used for summative assessment.

Result

More difficult to assess than student opinion is the objective mathematical attainment resulting from the adoption of HELM resources given that there were a number of changes made to address high failure rates for level one maths modules. These included changes to the syllabus (without dumbing down), changes to assessment, new module coordinators, new timetabling, class streaming, group projects and a better integration with the Universitys maths support unit. However, given the central role of the HELM workbooks in module delivery and the use CAA for formative feedback, it is worth crediting these in part for the much higher pass rates we have now been able to achieve.

In the year 2005-6, after the collation of phase test and exam marks 12% failed the module outright hence requiring re-sits. It should be noted the 57% passed all phase tests outright and bulk of the remaining students were able to demonstrate required achievement in the end of module exam. For comparison, in the year 2002-3 prior to module changes there was a 52% failure rate with significant impact on retention despite the offer of re-sits.

Although students have clearly benefited from changes to the structure of assessment, one can still infer significant improvements due to mathematics delivery and the role of HELM.

Although one could argue changes to assessment and subsequent higher pass rates dont necessary reflect an substantial improvement in the math prerequisite for later modules, there is some anecdotal evidence coming back from module coordinators of level two and three modules that the recent cohorts are better able to cope with the mathematics and better able to engage with student centred learning than in recent previous years.

Staff

HELM resources have proved especially attractive as means of delivering mathematics modules within science and engineering programmes and since their adoption in acoustics programmes module co-ordinators from other discipline areas in Computing, Science and Engineering at Salford (notably computing and civil / mechanical engineering) have expressed an interest. Clearly some discipline

areas have well established and tailored resources for mathematics delivery and even in acoustics it was deemed necessary to work in some specific acoustic examples to better justify the mathematic content to the students.

Dissemination of higher level workbooks within level 2 and 3 acoustics modules has proved difficult, given staff have well established lesson plans and perhaps fear the greater mathematical depth and lack of specific application might limit HELMs role in their lessons. However, as extra support materials, the resources are welcomed by both staff and students. The author has found it necessary to create password protected online access to resources for level 2, 3 and MSc with time dedicated to promotion of their use. Future policy may well include setting aside school funds and time to printing a selected set of advanced topic workbooks and assigning different module coordinators to distribute these in classes that have clear relevance (e.g. z-transforms in Digital Signal Processing).

Conclusion

HELM resources have played a central role in the delivery of mathematics to students doing acoustic related degree programmes in the department of Computing, Science and Engineering at the University of Salford. We can verify that the inclusion of HELM resources has been popular with students and has made a significant contribution to 'Level 1' pass rates, student mathematics attainment and more generally student learning skills. It should be noted that we suggest that to optimise the effectiveness and popularity of HELM workbooks and CAA module coordinators should consider restructuring lessons plans to make HELM central as opposed to supplementary to student learning of mathematics.

HELM Workbook
Structure and Notation $\boxed{50.6}$

1. List of Workbooks

No.	Title	Pages
1	Basic Algebra	89
2	Basic Functions	75
3	Equations, Inequalities & Partial Fractions	71
4	Trigonometry	77
5	Functions and Modelling	49
6	Exponential and Logarithmic Functions	73
7	Matrices	50
8	Matrix Solution of Equations	32
9	Vectors	66
10	Complex Numbers	34
11	Differentiation	58
12	Applications of Differentiation	63
13	Integration	62
14	Applications of Integration 1	34
15	Applications of Integration 2	31
16	Sequences and Series	51
17	Conics and Polar Coordinates	43
18	Functions of Several Variables	40
19	Differential Equations	70
20	Laplace Transforms	73
21	z-Transforms	96
22	Eigenvalues and Eigenvectors	53
23	Fourier Series	73
24	Fourier Transforms	37
25	Partial Differential Equations	42
26	Functions of a Complex Variable	58
27	Multiple Integration	83
28	Differential Vector Calculus	53
29	Integral Vector Calculus	77
30	Introduction to Numerical Methods	64
31	Numerical Methods of Approximation	86
32	Numerical Initial Value Problems	80
33	Numerical Boundary Value Problems	36
34	Modelling Motion	63

No.	Title	Pages
35	Sets and Probability	53
36	Descriptive Statistics	45
37	Discrete Probability Distributions	60
38	Continuous Probability Distributions	27
39	The Normal Distribution	39
40	Sampling Distributions and Estimation	22
41	Hypothesis Testing	42
42	Goodness of Fit and Contingency Tables	24
43	Regression and Correlation	32
44	Analysis of Variance	57
45	Non-parametric Statistics	36
46	Reliability and Quality Control	38
47	Mathematics and Physics Miscellany	70
48	Engineering Case Studies	97
49	Student's Guide	57
50	Tutor's Guide	143

2. Nature of the Workbooks

The 50 HELM Workbooks comprise:

- 33 Mathematics Workbooks (1 to 33) written specifically with the typical engineering student in mind containing mathematical topics, worked examples, tasks, exercises and related engineering examples.

- 1 Workbook (34) emphasising the mathematical modelling of motion, with worked examples, tasks, exercises and related engineering examples.

- 12 Statistics Workbooks (35 to 46) written specifically with the typical engineering student in mind containing statistical topics, worked examples with an emphasis on engineering contexts, tasks and exercises.

- 1 Workbook (47) containing Sections on Dimensional analysis, Mathematical explorations and 11 Physics case studies.

- 1 Workbook (48) containing 20 Engineering Case Studies ranging over many engineering disciplines.

- 1 Students' Guide (49) containing helpful advice, various indexes and extensive facts and formulae sheets for mathematics and mechanics.

- 1 Tutor's Guide (50) relating success stories and challenges and encapsulating good practice derived from trialling in a variety of institutions with their individual contexts and cultures.

The Workbooks are subdivided into manageable Sections. As far as possible, each Section is designed to be a self-contained piece of work that can be attempted by the student in a few hours. In general,

a whole Workbook represents about 2 to 3 weeks' work. Each Section begins with statements of prerequisites and desired learning outcomes.

The Tasks include space for students to attempt the questions; many Tasks guide the student through problems in stages.

It is often feasible for the lecturer to select certain Sections from a Workbook and omit others, reducing the reproduction costs if using hardcopy and better tailoring the materials to the needs of a specific group.

3. Notation Used

Fonts

In general HELM uses italic serif font letters to represent functions, variables and constants. However, as exceptions HELM Workbooks use the following non-italic sans-serif letters:

Mathematics

e for the exponential constant and for the exponential function (primarily use in introductory Workbook 6, elsewhere e is often used)

i where $i^2 = -1$

ln for natural logarithm

Statistics

E for Expectation   P for Probability   V for Variance   M for Median

Complex numbers

HELM uses i rather than j to represent $\sqrt{(-1)}$ so $i^2 = -1$, although there are one or two exceptions to this (in Workbook 48: Engineering Case Studies).

Vectors

HELM uses underlining of vectors rather than using bold e.g. $\underline{a}$

HELM uses $\underline{\hat{n}}$ for the unit normal vector but does not put the ˆ on the basic unit vectors in the x, y and z directions which have the standard symbols $\underline{i}, \underline{j}, \underline{k}$.

Identities

Although HELM introduces and uses the identity symbol '$\equiv$' extensively in Workbook 1: Basic Algebra and in Workbook 4: Trigonometry it is not normally used elsewhere and the more normal '$=$' is used except where emphasis seems advisable. (HELM is therefore not consistent.)

Issues and Notes for Tutors

1. The use of units in applied problems

Problems in engineering almost invariably involve physical quantities, for example distance, mass, time, current, measured in a variety of units: metres, miles, amps, litres, etc. When using mathematics to solve applied problems we often are a little slipshod in our approach to the way unknown variables and associated units are introduced and used.

We may ask for the solution of the equation $\sin x = 0.5$ where x, measured in radians, is in the interval $0 \leq x < 2\pi$. But should we say

(a) "Solve the equation $\sin x = 0.5$ where x, measured in degrees, is in the interval $0 \leq x < 360$", or

(b) "Solve the equation $\sin x° = 0.5$ where x is in the interval $0 \leq x < 360$", or

(c) "Solve the equation $\sin x = 0.5$ where x is in the interval $0° \leq x < 360°$"?

Purists may prefer (b) of these options.

Now consider the following problem.

A train is travelling at 80 km h^1 and is 2 km from the next station when it starts to brake so that it comes to a halt at the station. What is the deceleration, assuming it to be uniform during the time that the brakes were applied?

Here is the 'correct' solution.

Let the deceleration be a km h^{-2}.

In 2 km the speed of the train will have decreased by 80 km h^{-1}.

We use the formula $v^2 = u^2 - 2as$, where u km h^{-1} is the initial speed,

v km h^{-1} is the final speed and s km is the distance travelled.

Hence $= \dfrac{u^2 - v^2}{2s} = \dfrac{80 \times 80 - 0 \times 0}{2 \times 2} = \dfrac{80 \times 80}{2 \times 2} = 1600.$

The (uniform) deceleration is therefore 1600 km h^{-2}.

In this approach, a represents the **magnitude** of the deceleration and is a pure number.

A less 'professional' solution now follows.

Let the deceleration be a.

In 2 km the speed of the train will have decreased by 80 km h^{-1}.

We use the formula $v^2 = u^2 - 2as$ where u is the initial speed, v is the final speed and s is the distance travelled.

Hence $a = \dfrac{u^2 - v^2}{2s} = \dfrac{80 \times 80}{2 \times 2} = 1600$ km h^2.

In this second approach, a represents the deceleration itself and so has units. It seems simpler and maybe more natural. However, we then get the inconsistency that the fraction $\dfrac{80 \times 80}{2 \times 2}$, a number, is equated to a deceleration a, which is nonsense.

In the HELM Workbooks we have not always given rigorous solutions or taken the greatest care over the use of units, taking instead a pragmatic line and trying to keep solutions simple.

In truth, in the HELM Workbooks we have not been consistent in our approach, adopting whatever seemed appropriate to the situation.

We hope that the lecturer will understand what is required for a rigorous solution and forgive us these lapses. (Any reports of errors or suggestions for improvement of any HELM resources will always be welcome.)

2. Confidence

We want engineering students to be confident in their mathematics. However, this needs to be justified confidence. Some informative research by Armstrong and Croft (1999) included a diagnostic test of mathematics for engineering undergraduates and a confidence test. The results across 28 different mathematical topics (ranging from arithmetic to integration) were illuminating: There was not a single topic for which the proportion of students expressing a lack of confidence matched or exceeded the proportion of students answering wrongly. In other words there was **overconfidence across all topics**.

It can be concluded that in general any student seeking help needs it and so will many others who may not admit to it! A student telling his tutor that he has no problems should not be assumed to be correct - some gentle probing is always advisable.

Reference: Armstrong P.K. & Croft, A.C., (1999) *Identifying the Learning Needs in Mathematics of Entrants to Undergraduate Engineering Programmes in an English University*, European Journal of Engineering Education, Vol.24, No.1, 59-71.

3. HELM Workbooks and Students with Special Learning Difficulties

The HELM project sought expert advice on the style of the Workbooks. This included some interviews with selected students. In most cases HELM was able to respond to make the Workbooks (based on the earlier Open Learning Project) more accessible to all students.

When tested with various formats, most students with learning difficulties such as dyslexia preferred a sans serif font version and this has been adopted. However, a serif font is used for the mathematical notation. It seems, from past experience that this is preferable since the sans serif mathematical notation is unfamiliar and ambiguous giving, for example a straight x instead of curved, easily confused with a times sign, and the digit one looking like the upper case letter "i" and lower case letter "l" and also like the modulus sign.

Although there is an occasional use of italicised words, the preference is to use emboldening, which is clearer.

The main suggestion which was not followed concerns text justification. The recommendation is to use 'ragged right' text. Although fully justified text can cause problems for some students it was decided that HELM's use of very short paragraphs made that a less important consideration and full justification improved the general appearance of the page and emphasised the blocks of text.

A feature of the original Workbooks was that all answers and solutions to student Tasks and Exercises were printed upside down. Although this feature was welcomed by staff, the practice was roundly condemned by many students (and by Special Needs advisers too!), and so was abandoned.

It is intended to provide an alternative version with the solutions omitted.

4. Dyscalculia and Dyslexia Interest Group (DDIG)

In January 2003, the Dyscalculia and Dyslexia Interest Group (DDIG) was established. It began as a local interest group, based in the Midlands area of the UK. The group brings together both Mathematics support and dyslexia support tutors, from higher education, and is concerned with supporting the mathematical needs of dyslexic and dyscalculic students.

Since its inception DDIG has grown rapidly and there has been considerable interest in the group's work and members represent many different institutions. The group meets several times a year to share experiences and ideas, to exchange information, offer a forum for discussion of issues arising and link research and practice, and create a pool of resources. DDIG hold a series of events such as seminars, workshops and exhibitions, which aim to promote examples of good practise and raise awareness of these issues. The DDIG web site is:

http://ddig.lboro.ac.uk

which shows the aims of the group, events, contact information, information for tutors and students, useful reading, useful websites and research interests.

5. National Co-ordination Team For Widening Participation

The Action on Access website is a user-friendly site offering a wide range of resources, materials and information about, among other things, disability issues in higher education.

Their website is:

http://www.actiononaccess.org/

As at June 2006 there were 15 resources specifically related to the Maths, Stats and OR Network (see Resources Directory).

6. Student Difficulties

On the next two pages are reproduced two actual engineering students' attempts at an algebra problem and a calculus problem. They demonstrate a range of errors.

These examples can be used as a basis of discussion with tutorial assistants, new staff and others as to how to diagnose and remedy problems experienced by students challenged by mathematical problems.

You will be able to amass your own collection of such examples . . .

An Engineering Student's Errors in Algebra

Below is the work of a first year undergraduate Manufacturing Engineering student.

1. Study the student's work.

2. Note down the different kinds of mistakes made.

3. How would you help this student?

1. $144^{\frac{1}{2}} + 144^{\frac{3}{2}} = 144^{\frac{3}{4}}$

2. (a)

$$
\begin{aligned}
(x+6)(x+5) &= (6+x)(5+x) \\
&= 6(5+x) + x(5+x) \\
&= 30 + 6x + 5x + x^2 \\
&= x^2 + 11x + 30
\end{aligned}
$$

(b)

$$
\begin{aligned}
(x+1)(x+2)(x+3) &= (x^2 + 2x + x + 2)x + 3 \\
&= (x^2 + 3x + 2)x + 3 \\
&= x^3 + 3x^2 + 2x + 3x^2 + 9x + 6 \\
&= x^3 + 6x^2 + 11x + 6
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
11x + 7 &= 0 \\
11x + 7 - 7 &= -7 \\
\frac{11x}{11} &= \frac{-7}{11} \\
x &= \frac{-7}{x}
\end{aligned}
$$

(b)

$$
\begin{aligned}
x^2 + 6x - 7 &= 0 \\
x^2 + 6x - 7 + 7 &= 7 \\
x^2 + \frac{6x}{6} &= \frac{7}{6} \\
x^2 + x &= \frac{7}{6} \\
x^3 &= \frac{7}{6} \\
x &= \frac{7}{18}
\end{aligned}
$$

(c)

$$
\begin{aligned}
x^2 + x - 8 &= 0 \\
x^2 + x - 8 + 8 &= 8 \\
x^3 &= 8 \\
x &= \frac{8}{3}
\end{aligned}
$$

An Engineering Student's Solution to a Calculus Problem

Below is the work of a first year undergraduate Electrical Engineering student.

1. Study the student's work.

2. How many errors can you see?

3. How would you classify each of them?

4. How could you help this student?

Problem

Given:

$$y = \left(\frac{x+7}{4x-3} \right)^{\frac{1}{3}}, \qquad x > \frac{3}{4}$$

find the gradient at $x = 1$.

Student's Solution

$$\ln y = \frac{\ln(x+7)^{\frac{1}{3}}}{\ln(4x-3)^{\frac{1}{3}}}$$

$$\ln y = \frac{\frac{1}{3}\ln(x+7)}{\frac{1}{3}\ln(4x-3)}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\dfrac{\ln(4x-3)}{x+7} - \dfrac{\ln(x+7)}{4x-3}}{(\ln(4x-3))2}$$

$$\frac{dy}{dx} = \left(\frac{x+7}{4x-3} \right)^{\frac{1}{3}} \times \left(\frac{\dfrac{\ln(4x-3)}{x+7} - \dfrac{\ln(x+7)}{4x-3}}{(\ln(4x-3)^2} \right)$$

When $x = 1$
$$\frac{dy}{dx} = \left(\frac{8}{1} \right)^{\frac{1}{3}} \times \left(\frac{\dfrac{\ln 1}{8} - \dfrac{\ln 8}{1}}{(\ln 1)^2} \right)$$

$$= 2 \times \frac{0-0}{0}$$

Therefore
$$\frac{dy}{dx} = 0 \quad \text{when} \quad x = 1$$

HELM Computer Aided Assessment

<div style="text-align: right;">**50.8**</div>

1. Introduction

The HELM project's strategy for addressing the practical difficulties currently encountered in the teaching of mathematics to engineering undergraduates, is to produce and disseminate high quality teaching and learning materials supported by a regular CAA testing regime.

The banks of CAA questions developed by the HELM project presently contain around 5000 questions. The questions are held in Question Mark Perception version 3.4.

Historically the HELM project has developed its CAA resources based on earlier work of the Loughborough Open Learning Maths Project which predated the trouble-free use of MathML. Following the OLMP chosen path, the body of each question and solution in the HELM CAA databank is presented as a jpg image, captured from a pdf file derived from an original Latex file. The MathML approach was considered but, in view of the additional expertise required to develop questions and the need for users to have MathML enabled browsers, its use was not deemed to be ideal at the time the HELM project started in 2002.

2. Underlying Structure of CAA Questions

HELM Questions have been designed to match particular mathematical concepts in support of the topics covered by the HELM workbooks.

Within QM Perception a consistent naming convention has been adhered to which clearly identifies the location of the topic within the workbook structure and describes the question so that its purpose is readily discernible.

For example, Cramer's rule for solving a system of 3 by 3 simultaneous equations is covered in Workbook 8 in Section 8.1, subsection 2 and the associated CAA question library is named as: fn_08_01_02_cramer_3x3, where the "fn" indicates that the question library is nominally formative and contains questions which are numeric in nature.

In a similar way, a summative, multiple choice library would have "sm" at the start of its name and a formative library containing matrix style questions would be designated "fma".

The questions relevant to each concept have been structured, as indicated above, into two sets, one nominally designated **formative** (F) the other **summative** (S). With very few exceptions, each set contains 10 questions cloned from a designated single master question, thereby ensuring that the same level of difficulty is maintained, and justifying the random selection of questions from each set

for test purposes. Several concepts may be selected and appropriate questions chosen randomly and presented within QM Perception as a customised test.

Question feedback is an option, which may be enabled in both formative and summative type questions, but we strongly recommend that it always enabled with formative questions as a motivational and pedagogical aid which drives student learning.

In the HELM questions clicking the question number after submission shows the original question, the learners response, and the HELM feedback in a scrolling format. In many cases this feedback shows the specific solution or an example solution while in simpler cases a generic solution may be presented. The importance of providing specific feedback for the benefit of the weaker learner is illustrated by the following student comments:

- "I wish the practice coursework tests gave better feedback, i.e. step by step showing with actual values rather than just partial workings out with algebra, especially for types of questions not found in the HELM exercise books."

- "I believe the feedbacks from the practice test can be improved by giving fully explanations to how to do it and along with working answer. From my past experiences, some of the questions I weren't able to answers (usually the harder ones) were lack of explanations and no working answer to the problem."

3. Types of Question Used

From the authoring point of view, the simplest response required to a particular CAA question is the input of a numerical value, which may be either a whole number or a decimal.

An advantage of this Numeric entry type of question is that it is simple to construct and allows for the easy generation of clones with which to populate the relevant question library. Also, when answers are requested to an accuracy of, say, 2 decimal places, it is not likely that learners will be able to guess the correct response.

However, there are disadvantages to the use of the QM Perception based HELM Numeric entry type question that need to be addressed, particularly with regard to the accuracy required for the response, and the issue arises as to exactly what is being tested. It is a common occurrence for a learner to understand all of the mathematics related to a question but simply fail to round their numeric answer correctly at the last stage where for example 2.136 required to 2 decimal places is entered as 2.13, or, more subtly, 2.1346 is firstly represented as 2.135 and then is entered as 2.14.

The predecessor to HELM accepted precise answers only, believing it to be important that engineering undergraduates understand the need for precision. However, feedback from students indicates that this policy was a matter of some concern to them, particularly when taking summative tests:

- "With the CAA tests, being off the answers by 0.01 can result in your answer being incorrect, causing you to lose a lot of marks even though your method and approach are correct. I think it would be better to have a range of answers for questions that require you to round off your answer."

- "The online CAA tests do not take account of an understanding of the subject matter, only an ability to produce an exact answer required. This does imply understanding, but making an error to a 100th does not imply misunderstanding."

- "Like others - I have strong queries about the CAA. In one test I had all of the correct working, but got an answer which was 0.01 out due to rounding. I received no marks. I have done this in a few tests and am upset and concerned about my marks."

In an attempt to address this problem HELM questions allow for responses within a tolerance where rounding is required, for example, ± 0.01 for questions requiring 2 d.p. accuracy.

We attempt to alert learners to this source of error by indicating in the feedback to them that, while their answer has been allowed, there is the possibility that they have made a rounding error.

The CAA questions as written do not normally penalise the learner for this type of error, but the opportunity to exact a penalty is available at the discretion of the user.

HELM numeric type questions now accept up to four possible inputs, for example the real and imaginary parts of two complex solutions to a problem.

The ability to accept and mark multiple numerical answers goes some way to allowing credit for separate parts of an answer, and may compensate for the occasional careless error, by rewarding the remaining correct responses.

In some circumstances a Numeric response is inappropriate, for example in the factorising of a quadratic expression. In such circumstances the Multiple Choice type of question has been used.

This approach has the advantage that it avoids the earlier Numeric input issue, but is of course susceptible to guesswork. A further requirement of this type of question is the construction of realistic, and not obviously wrong, distracters, preferably based on knowledge of typical errors and misconceptions. As a consequence, this type of question is more difficult to produce, and is especially challenging when many clones are needed.

Other options used very occasionally are the Matrix and Drop Down List type of question.

4. Multi-Stage Questions

A disadvantage inherent in single stage questions (where the final answer only is expected) is that a wrong response does not give credit for any correct work that might have been done by learners prior to submission of their answer. In some questions where several processes may have been required before the final result is obtained, the loss of all credit seems unfair, and is again an issue that has been commented upon by students:

- "The tests are not a representation of my understanding. Just number plugging. You can be 99.99% correct but gain no marks. There should be a tolerance in marking, as no marks for methods can be achieved."

- "The tests would be far better if they took into account working out."

In an attempt to address this situation some questions have been written in Multi-Stage format, whereby credit is given for a correct response at each of several stages within a question. The learner, having perhaps submitted an incorrect answer at an intermediate stage, is subsequently presented with sufficient information at the commencement of the next stage to allow him or her to continue, thus giving the opportunity to gain partial credit within a more complex question. As one student commented:

- "The computer tests should be developed so that marks for workings can be given. Currently it is very easy to obtain low marks, despite having a good grasp of the subject."

Another simpler possibility is to link separate question libraries by a common theme, so that in their entirety they cover the techniques required for the solution of a more complex problem. For example, the solution of a second order differential equation might be covered by libraries which separately test the concepts of (a) forming the auxiliary equation, (b) solving such an equation, (c) interpreting the solution and selecting an appropriate complementary function, (d) selecting the general form of a particular integral, (e) using initial conditions.

This also allows for credit to be given independently to each process involved in the solution of this more complex type of problem.

5. HELM CAA Regime

Mathematics learners within Loughborough University Engineering Faculty typically take 2-5 summative computer based tests within a semester for a mathematics module. The testing regime allows learners to take formative tests for a period of several days before the summative test is made available. These formative tests are structured in exactly the same way as the summative test and there is no limit to the number of times that students may take these practice tests.

A test covers, say, 10 concepts, with a question randomly chosen from perhaps 10 questions available in each library. This provides a determined student with the possible opportunity to attempt 100 different questions, and to study question specific feedback prior to formal assessment.

Since students know that the summative questions are of a similar nature they are well motivated to undertake the formative tests and so gain confidence (and competence) prior to summative testing.

Analysis of student logs shows intensive activity during this practice test period. Feedback from students demonstrates how much this aspect of the assessment is appreciated:

- "The trial tests are a really helpful tool to mastering the subject at hand. Many people would just revise for the test and not learn a great deal, but by having practice tests it makes it a less formal way to prepare for the test but is also aiding revision for the module. Should definitely be kept."

6. Dissemination of the HELM CAA Regime

An engineering academic using HELM resources said:

- "Students undertake two phased tests per semester, which are paper based and supervised. However, the HELM CAA system is used to provide trial tests to help the students prepare for the formal tests. These tests are available online and students need to login to access them." (Salford University)

For HEIs in England and Northern Ireland, HELM can provide question banks in the form of a QMP "dot question" file for copying into the database folder of QMP Question Manager, together with the associated graphic images in a zipped format, and the required templates. For institutions who are not using QMP we will provide some predefined tests on a stand alone CD ROM.

From a staff perspective, CAA testing allows for monitoring of student understanding at intervals within the teaching period via the reporting facility within QM Perception. Identification of misunderstood concepts is thus possible and enables remedial action to be taken.

Within such a flexible testing regime, allowing students the opportunity to take **unsupervised** summative tests carries some risk. In line with the requirements of the code of practice for the use of information technology (IT) in the delivery of assessments (2), we must be confident that our assessments are reliable and fair. BS 7988 (3) is relevant to this:

- "Colleges, universities, schools and businesses are increasingly using computers to deliver exams and assessments in place of pen and paper. This standard (BS 7988) will set guidelines to follow so that students know they will be treated fairly and so that organizations can trust and rely on the results of computer-delivered assessments." (John Kleeman of Question Mark Computing).

The following student comment reveals that this may not be the case in spite of clear guidelines which are issued to students prior to taking the summative test:

- "Many students also take the tests together, making it hugely unfair for students who follow the guidelines."

- "I do also feel that many on my course are sitting this test together to aid their "learning" and results. I do not do this and I feel the marks I gain are my own, perhaps if I want higher marks I should start doing what is perceived as "normal" in the student body and effectively cheat by sitting the test with others' support. I wish to make you aware that a good deal of the students (if I was to rough a guess - maybe 60%) are gathering to take these tests - It is not fair on the rest of us who only have on mind working on these tests."

It would seem to be necessary to design into a formal testing system a mechanism for **invigilation** if this unfair practice is to be eliminated. Such trials on a small scale have been undertaken at Loughborough in 2004-5 with success. Problems which could arise if widely adopted are:

- time burden on staff

- lack of sufficient computer laboratory spaces

- overload on the server delivering the test

- logistical problems should a timed test fail

HELM practice tests do not need supervision as their main purpose is to drive learning through continuous formative assessment, where collaboration, discussion and access to external resources are to be encouraged. However, where summative testing is significant then careful thought is needed when determining the assessment regime.

7. Further Details of Loughborough's Implementation of the Assessment Regime

In a typical testing regime students are given a Workbook for a new topic in Week 1 (for self-study or lecture support). Week 3 is then a Test week (during which lectures and tutorials run as normal, but on a new Workbook). The Test week is organised as follows:

- On Monday to Wednesday a practice test is available on the web. Students may access this test at any time within this period and, as it is web-delivered, anywhere in the world. It can be taken as often as the student desires. No record is kept by staff of student performance on these trial tests (although it could be - for research and development purposes, for example). Some students simply access the trial test to get information on question types and level of difficulty without attempting to answer any questions. Most will make a serious attempt at the trial test at least once; many up to five times. Many will work in small groups sorting out difficulties with the trial test. A good number seek help with the trial test from staff in our Mathematics Learning Support Centre. Others will access the test, input spurious answers just in order to get the feedback or possibly to try to discover all the possible questions! We find 95% of students engage in some way. This is a valuable learning mechanism and it is clear that students now engage with the learning process at some level throughout the semester.

- On Thursday and Friday the actual coursework test is available. Again, students may access this test at anytime within this period and from anywhere. However, they are only allowed to take this test once.

Both tests have an identical form, selecting questions randomly from previously created question libraries covering aspects of the topic just covered in lectures. If a student gets a question wrong on the practice test a single page of feedback is available. The feedback may be exemplary (addressing the solution of similar problems to the one presented) or specific (in which the solution to the given problem is detailed). The only feedback currently available on taking a formal coursework test is the overall score and an indication of which questions were answered correctly and which incorrectly. However, HELM CAA questions now contain feedback in both summative and formative libraries.

The provision of feedback in a summative test may be switched off within QMP but it should be noted that the learners score and indications of correctly answered questions will also not be presented. The option to switch off all feedback may well be desirable as a mechanism for curbing possible misuse by students who wish to collude.

Although there are many possible question types we generally use just two: numeric input (the majority) and multiple choice. Multiple response questions will be used in the future.

Following extensive feedback exercises we find this testing regime to be generally popular with both staff and students. Students particularly like the flexibility this method of assessment offers. They like the facility to practice tests and the possibility of doing tests when they are ready. A recurring complaint about this approach is its unforgiving nature: if the final step is incorrect then no marks are awarded even if every other step was accomplished correctly. However CAA questions are relatively straightforward and so we would expect students to get them right each time. It also encourages careful working, which is no bad thing. Newer types of questions, allowing for multiple responses, staged responses and ranges of tolerance have now been incorporated which should go some way to mitigating such student concerns.

An occasional and valid concern raised by academics is the current practice at Loughborough of using the same libraries for both trial testing and formal testing. In the latest version of the database questions are held in separate libraries by which means we expect to discourage a rote-learning approach by students. Another legitimate concern is over allowing Loughborough students to undertake the formal tests unsupervised. There are great benefits to the students to allow them this freedom but at least some supervised tests, and careful monitoring of server log records for multiple coursework test log-ons would seem wise. This is very much up to the individual academic or institution to decide upon.

8. Implementation at other Institutions

The HELM computer aided assessments (CAA) have been constructed using Question Mark Perception (v3.4) and upon request the HELM question database will be freely distributed in this format to those institutions who wish to make use of it. For those institutions without QMP a standalone disc containing some pre-determined sample assessments has also been created.

The project recognises that there is interest in other vehicles for delivery of CAA and is grateful for the cooperation provided by TOIA (Technologies for Online Interoperable Assessment) in examining the possibility of using the HELM questions within the TOIA assessment engine.

With the collaboration of our consortium partners at the University of Hull small scale trials, hosted on TOIA's Strathclyde server, were conducted using an earlier form of the HELM question database limited to single input numeric (without range) and multiple choice question formats. The subsequent report noted that while the online system worked well there were initially some communication issues related to the use of the remote server and that a fairly steep learning curve was experienced by staff using the system.

We also would like to thank Oxford Brookes University for undertaking trials of HELM questions

within WebCT and for providing information of the procedure adopted to convert QTI questions sent from the HELM Project into databases for WebCT.

The numerical question types now in use by HELM allow for up to four numeric values to be entered within a single question and also provide for answers to be marked as correct when falling within a particular range.

For those institutions who may wish to investigate the feasibility of importing HELM questions into other delivery systems such as TOIA or WebCT the following points should be verified.

- Answers are being correctly marked.

- Layout and labelling of multiple input numeric answer boxes is not corrupted.

Regarding the marking of questions:

QMP (v3.4) ignores trailing zeros and will therefore consider numbers such as 2.3 and 2.30 to be identical and mark 2.3 as correct when 2.30 is specified as the answer (and vice versa). Other assessment engines may not be as tolerant!

Also, within QMP (v3.4) an integer answer specified as for example "3" will not only allow "3.0" to be marked as correct but also "3.1", "3.2", etc.

In order to prevent this marking error occurring, questions requiring integer answers such as "3", have been specified within HELM as requiring answer "3.0", which allows "3" to be correctly marked but prevents the aforementioned error from occurring.

Regarding layout and labelling:

Answer boxes in some questions have been positioned in a form other than the QMP default and also labelled. If importing HELM questions using QTI XML these formats may be corrupted.

Neither of the above are issues for QMP users who will be provided with the *helm.question* **file which can be dropped directly into the QMP database folder.**

HELM Electronic Learning Resources

1. Introduction

HELM has 50 Workbooks and 80 Interactive Learning Segments and linked Revision Questions (with inbuilt randomisation).

The Interactive Learning Segments provide web-based versions of some Sections of many of the more elementary Workbooks and contain animations to generate interest and have linked Revision Exercises where randomly generated questions are provided for the benefit of students working independently.

These Interactive Learning Segments and linked Revision Exercises have been found to be especially useful for supporting students who find it difficult to cope with the mathematical demands of their programmes.

The animations are also useful for illustrating lectures and for revision.

The tutor will provide guidance as to how the materials are to be used.

[Reproduced with minor amendments from the Student's Guide]

2. Accessing HELM electronic learning resources

The web based versions of HELM Interactive Learning resources can be accessed via

http://helm.lboro.ac.uk/cal/

or via any specific web address (url) given to you by your tutor.

Once you access this web page, you will see four links as shown below:

Workbooks   Interactive Lessons   Revision Questions   Help

Clicking on either an icon or the hypertext link below the icon will take you to the corresponding web page containing links to the selected learning resources.

Workbooks:

There are fifty HELM Workbooks available to HEFCE-funded Higher Education Institutions in England and Northern Ireland from the Mathematics Education Centre at Loughborough University.

Access to these Workbooks is restricted to staff and students of these institutions and is controlled by each institution.

The List of Workbooks and Sections and some samples can be viewed on the HELM Website.

Interactive Lessons and Revision Questions:

HELM Interactive Lessons and Revision Questions are Authorware 7 applications. Prospective users need to have the appropriate Macromedia$^{®}$ Authorware$^{®}$ player (a web browser plug-in) installed in order to use these. The Authorware$^{®}$ player could be downloaded free of charge from:

> http://www.adobe.com/shockwave/download/alternates/ #ap (as at June 2006)

It is important to **choose the correct version of the player** to support the browser and the computer platform being used. See the on-screen help file (by clicking HELP icon or the hyperlink) for further details.

Only selected sections of Workbooks 1-20 and 35-39 have associated Interactive Lessons and Revision Questions.

Caveat: Most of the Interactive Lessons and Revision Questions are derived from Loughborough's Open Learning Project, which predated the HELM project, so the Workbook text may differ from that in the corresponding HELM Workbook.

Once you are in the page giving links to available lessons or questions, select the link you wish to continue with. An example for a lesson introduction page is given below.

The introduction page presents you with the title of the Workbook and the section that is covered by the resource you selected. It will also state the prerequisites and the learning outcomes.

Click on the button labelled "Enter" to launch the Interactive Lesson or the Revision Questions. Provided that the correct web player plug-in is installed, the introduction page to the Interactive Lesson or the Revision Questions chosen will be displayed within the web browser. Patience may be needed as in some cases it may take a few seconds to load the application on to your web browser. The front page of the lesson will look similar to the following example.

As you see, there may be one or more buttons that will take you to a particular subsection of the lesson. For example, referring to the figure given below, clicking on the top button would take you

to a section on Parametric differentiation; clicking the bottom button would take you to a section on Higher Derivatives.

⇨ Parametric differentiation

⇨ Higher derivatives

After you click on one of the buttons, the front page will disappear and you will see the first page of your chosen section. Look at the top right-hand corner of the screen: inside a box you will see in **green** something like "Page 1/2". This means that the section contains two pages of material and that you are on the first page. Note that a page can contain several screens of material.

The learning material is multimedia: a mix of text, graphics and sound. We think that the audio aspect of the material is useful, so you should use headphones if at all possible. The material is also interactive: most sections contain questions that you are expected to answer before proceeding. These questions might require you to use the mouse - to draw a vector, for instance. More often, they will require you to enter a number as the answer to a calculation. So be prepared: when you sit down at the computer make sure that you have with you a pen, some paper and a calculator.

Follow the instructions on the screen. Often, this means that you are presented with some text (maybe with some graphics) and then have to press the continue button in order to see more text. You will gain most from this activity if you think about the material. Try to anticipate what will come next; try to complete the next stage in an argument before it is presented to you; if you are asked to do something before proceeding - do it. Don't just sit there and idly press the continue button!

When a page is completed, the box at the top right-hand corner of the screen will turn **red**. You can then either navigate to the next page in the section, or return to the front page to choose another section. Or perhaps you want to quit the lesson altogether. These tasks are accomplished by using the navigation toolbar at the bottom right-hand corner of the screen and it is shown below. This toolbar is always visible. If you move your cursor over any icon in the toolbar, you will see a short pop-up message describing the icon's function. Once you have some experience with the system, this might prove irritating. You can turn off the pop-ups at any time by using the menu item entitled "Pop-ups", found at the top left-hand corner of your screen.

So that you have a permanent record, the icon functions are also described below.

To move to the next page of a section, click on: ⇨

To move to the previous page of a section, click on:

To return to the front page, click on:

To quit the lesson at any time, click on:

To move to the first page of a section, click on:

To move to the last page of a section, click on:

To get a list of previously viewed pages, click on: and then select one from the list to visit that page.

To search for a word or a phrase, click on:

In practice, the three navigation icons you will use most often are:

1. The icon that takes you to the next page

2. The icon that returns you to the title page

3. The icon that lets you quit.

Feel free to view each page of the Interactive Lessons as many times as you like. Note that many of the pages have random variables embedded within them, so that each time you view the page you see slightly different examples.

Similarly, you may attempt the Revision Questions as often as you wish. Nearly all of the Revision Questions are generated using random numbers so that you seldom get the same question twice. These questions are presented within the context of the theoretical material that is applicable to a particular Workbook. Once you have worked through a Workbook, though, you might want to gain more practice at doing questions without having to wade through the relevant background theory.

3. Description of HELM Workbook layout

On the following three pages are reproduced from the Student's Guide explanatory pages concerning Workbook Layout.

Description of HELM Workbook layout

Complex Arithmetic [10.1]

➡ Introduction

Complex numbers are used in many areas of engineering and science. In this Section we define what a complex number is and explore how two such numbers may be combined together by adding, subtracting, multiplying and dividing. We also show how to find 'complex roots' of polynomial equations.

A **complex number** is a generalisation of an ordinary real number. In fact, as we shall see, a complex number is a pair of real numbers ordered in a particular way. Fundamental to the study of complex numbers is the symbol i with the strange looking property $i^2 = -1$. Apart from this property complex numbers follow the usual rules of number algebra.

Workbook introduction.

What you should learn.

What you should know before you start.

➡ Prerequisites

Before starting this Section you should ...

- be able to add, subtract, multiply and divide real numbers

- be able to combine algebraic fractions together

- understand what a polynomial is

- have a knowledge of trigonometric identities

➡ Learning Outcomes

On completion you should be able to ...

- combine complex numbers together

- find the modulus and conjugate of a complex number

- obtain complex solutions to polynomial equations

Key Points.
Take especial note of these.

Key Point 1

The symbol i is such that

$$i^2 = -1$$

Using the normal rules of algebra it follows that

$$i^3 = i^2 \times i = -i \qquad i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$$

and so on.

Task for you to try with space for your working.
Answer presented after solution box.

Task

If $z = -2 + i$ and $w = 3 + 2i$ find expressions for

(a) $z + 2w$, (b) $|z - w|$ and (c) zw

Your solution

(a)

Answer

$z + 2w = 4 + 5i$

Your solution

(b) Hint: you should find that $z - w = -5 - i$

Answer

$|z - w| = \sqrt{(-5)^2 + (-1)^2} = \sqrt{26}$

Your solution

(c)

Answer

$zw = -6 + 3i - 4i + 2i^2 = -8 - i$

Worked example.
Solution with explanation follows in box.

Example 2
Find $\dfrac{z}{w}$ if $z = 2 - 3i$ and $w = 2 + i$.

Solution

$$\frac{z}{w} = \frac{2-3i}{2+i} = \frac{(2-3i) \times (2-i)}{(2+i) \times (2-i)} \qquad \text{rationalising}$$

$$= \frac{4 - 3 + i(-6-2)}{4+1} \qquad \text{multiplying out}$$

$$= \frac{1}{5} - \frac{8}{5}i \qquad \text{dividing through}$$

Exercise for you to do.
Answers follow in box (usually no detailed solution).

Exercises

1. Find the roots of the equation $x^2 + 2x + 2 = 0$.

2. If i is one root of the cubic equation $x^3 + 2x^2 + x + 2 = 0$ find the two other roots.

3. Find the complex number z if $2z + z^* + 3i + 2 = 0$.

4. If $z = \cos\theta + i\sin\theta$ show that $\dfrac{z}{z^*} = \cos 2\theta + i \sin 2\theta$.

Answers 1. $x = -1 \pm i$ 2. $-i, -2$ 3. $-\dfrac{2}{3} - 3i$

List of Sections in Workbooks 1 to 48

Workbook 6 - Exponential and Logarithmic Functions (73 pages)	
6.1	The Exponential Function
6.2	The Hyperbolic Functions
6.3	Logarithms
6.4	The Logarithmic Function
6.5	Modelling Exercises
6.6	Log-linear Graphs

Workbook 7 - Matrices (50 pages)	
7.1	Introduction to Matrices
7.2	Matrix Multiplication
7.3	Determinants
7.4	The Inverse of a Matrix

Workbook 8 - Matrix Solution of Equations (32 pages)	
8.1	Solution by Cramer's Rule
8.2	Solution by Inverse Matrix Method
8.3	Solution by Gauss Elimination

Workbook 9 - Vectors (66 pages)	
9.1	Basic Concepts of Vectors
9.2	Cartesian Components of Vectors
9.3	The Scalar Product
9.4	The Vector Product
9.5	Lines and Planes

Workbook 10 - Complex numbers (34 pages)	
10.1	Complex Arithmetic
10.2	Argand Diagrams and the Polar Form
10.3	The Exponential Form of a Complex Number
10.4	De Moivre's Theorem

Workbook 11 - Differentiation (58 pages)	
11.1	Introducing Differentiation
11.2	Using a Table of Derivatives
11.3	Higher Derivatives
11.4	Differentiating Products and Quotients
11.5	The Chain Rule
11.6	Parametric Differentiation
11.7	Implicit Differentiation

	Workbook 12 - Applications of Differentiation (63 pages)
12.1	Tangents and Normals
12.2	Maxima and Minima
12.3	The Newton-Raphson Method
12.4	Curvature
12.5	Differentiation of Vectors
12.6	Case Study: Complex Impedance

	Workbook 13 - Integration (62 pages)
13.1	Basic Concepts of Integration
13.2	Definite Integrals
13.3	The Area Bounded by a Curve
13.4	Integration by Parts
13.5	Integration by Substitution and Using Partial Fractions
13.6	Integration of Trigonometric Functions

	Workbook 14 - Applications of Integration 1 (34 pages)
14.1	Integration as the Limit of a Sum
14.2	The Mean Value and the Root-Mean-Square Value
14.3	Volumes of Revolution
14.4	Lengths of Curves and Surfaces of Revolution

	Workbook 15 - Applications of Integration 2 (31 pages)
15.1	Integration of Vectors
15.2	Calculating Centres of Mass
15.3	Moment of Inertia

	Workbook 16 - Sequences and Series (51 pages)
16.1	Sequences and Series
16.2	Infinite Series
16.3	The Binomial Series
16.4	Power Series
16.5	Maclaurin and Taylor Series

	Workbook 17 - Conics and Polar Coordinates (43 pages)
17.1	Conic Sections
17.2	Polar Coordinates
17.3	Parametric Curves

	Workbook 18 - Functions of Several Variables (40 pages)
18.1	Functions of Several Variables
18.2	Partial Derivatives
18.3	Stationary Points
18.4	Errors and Percentage Change

Workbook 19 - Differential Equations (70 pages)	
19.1	Modelling with Differential Equations
19.2	First Order Differential Equations
19.3	Second Order Differential Equations
19.4	Applications of Differential Equations

Workbook 20 - Laplace Transforms (73 pages)	
20.1	Causal Functions
20.2	The Transform and its Inverse
20.3	Further Laplace Transforms
20.4	Solving Differential Equations
20.5	The Convolution Theorem
20.6	Transfer Functions

Workbook 21 - z-Transforms (96 pages)	
21.1	z-Transforms
21.2	Basics of z-Transform Theory
21.3	z-Transforms and Difference Equations
21.4	Engineering Applications of z-Transforms
21.5	Sampled Functions

Workbook 22 - Eigenvalues and Eigenvectors (53 pages)	
22.1	Basic Concepts
22.2	Applications of Eigenvalues and Eigenvectors
22.3	Repeated Eigenvalues and Symmetric Matrices
22.4	Numerical Determination of Eigenvalues and Eigenvectors

Workbook 23 - Fourier Series (73 pages)	
23.1	Periodic Functions
23.2	Representing Periodic Functions by Fourier Series
23.3	Even and Odd Functions
23.4	Convergence
23.5	Half-range Series
23.6	The Complex Form
23.7	An Application of Fourier Series

Workbook 24 - Fourier Transforms (37 pages)	
24.1	The Fourier Transform
24.2	Properties of the Fourier Transform
24.3	Some Special Fourier Transform Pairs

Workbook 25 - Partial Differential Equations (42 pages)	
25.1	Partial Differential Equations
25.2	Applications of PDEs
25.3	Solution using Separation of Variables
25.4	Solutions using Fourier Series

Workbook 26 - Functions of a Complex Variable (58 pages)	
26.1	Complex Functions
26.2	Cauchy-Riemann Equations and Conformal Mappings
26.3	Standard Complex Functions
26.4	Basic Complex Integration
26.5	Cauchy's Theorem
26.6	Singularities and Residues

Workbook 27 - Multiple Integration (83 pages)	
27.1	Introduction to Surface Integrals
27.2	Multiple Integrals over Non-rectangular Regions
27.3	Volume Integrals
27.4	Changing Coordinates

Workbook 28 - Differential Vector Calculus (53 pages)	
28.1	Background to Vector Calculus
28.2	Differential Vector Calculus
28.3	Orthogonal Curvilinear Coordinates

Workbook 29 - Integral Vector Calculus (77 pages)	
29.1	Line Integrals Involving Vectors
29.2	Surface and Volume Integrals
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Workbook 30 - Introduction to Numerical Methods (64 pages)	
30.1	Rounding Error and Conditioning
30.2	Gaussian Elimination
30.3	LU Decomposition
30.4	Matrix Norms
30.5	Iterative Methods for Systems of Equations

Workbook 31 - Numerical Methods of Approximation (86 pages)	
31.1	Polynomial Approximations
31.2	Numerical Integration
31.3	Numerical Differentiation
31.4	Nonlinear Equations

Workbook 40 - Sampling Distributions and Estimation (22 pages)	
40.1	Sampling Distributions
40.2	Interval Estimation for the Variance

Workbook 41 - Hypothesis Testing (42 pages)	
41.1	Statistics Testing
41.2	Tests Concerning a Single Sample
41.3	Tests Concerning Two Samples

Workbook 42 - Goodness of Fit and Contingency Tables (24 pages)	
42.1	Goodness of Fit
42.2	Contingency Tables

Workbook 43 - Regression and Correlation (32 pages)	
43.1	Regression
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Workbook 44 - Analysis of Variance (57 pages)	
44.1	One-Way Analysis of Variance
44.2	Two-Way Analysis of Variance
44.3	Experimental Design

Workbook 45 - Non-parametric Statistics (36 pages)	
45.1	Non-parametric Tests for a Single Sample
45.2	Non-parametric Tests for Two Samples

Workbook 46 - Reliability and Quality Control (38 pages)	
46.1	Reliability
46.2	Quality Control

Workbook 47 - Mathematics and Physics Miscellany (69 pages)	
47.1	Dimensional Analysis in Engineering
47.2	Mathematical Explorations
47.3	Physics Case Studies

Workbook 48 - Engineering Case Studies (97 pages)	
	Engineering Case Studies 1 to 20

Commentaries on Workbooks 1 to 48

Workbook 1: Basic Algebra

The lack of fluency with basic algebra among engineering undergraduates is probably the area most commented upon by those who teach them mathematics and by engineering academics, who find that this deficiency hampers the student in solving problems in the wider engineering arena. To achieve the required level of fluency it is necessary to build the student's confidence by a careful programme of drill and practice examples. This idea may not be as fashionable as it once was, but effort invested with this material will reap rewards in later work.

This Workbook is therefore placed first in the series to emphasise the importance of the mathematics it covers. The five areas that it deals with are: mathematical notation and symbols, indices, simplification and factorisation (of expressions), algebraic fractions, and formulae and transposition.

Section 1.1 may seem elementary to the majority of students, but experience has shown that many of them have lacunae in their knowledge and understanding, even at this level. Particular attention needs to be paid to the understanding of, and correct use of, the modulus sign and the sigma notation. In the interpretation of index notation, care is needed with negative powers on the denominator of a fraction and the (obvious) fact that different powers of the same variable (e.g. $a^2 + a^3$) cannot be combined together. Correct use of a pocket calculator when fractional powers are involved also needs attention. In general it must be explained that using a pocket calculator successfully needs a good grasp of the underlying algebra.

The importance of Section 1.3 cannot be overstated. It is disappointing to find that so many undergraduates have difficulty in correct manipulation of algebraic expressions. To see in a student's attempted solution of a problem in reliability the 'mathematics' $1 - (1 - e^{-t}) \equiv e^{-t}$ demonstrates a lack of knowledge of the distributive law. Such errors are all too common and prevent the correct solution of straightforward problems in engineering. There are no short cuts: the basics must be thoroughly understood and students must be made aware of this. Factorisation should become second nature and this, too, demands time and effort.

Handling algebraic fractions is made more difficult these days when school students are not given a thorough grounding in the arithmetic of number fractions. It would be wise to give students some revision in dealing with number fractions as a precursor, so that the 'rules' for manipulating algebraic fractions make sense. Students who attempt to apply the rules without understanding them sometimes come a cropper, and subsequent stages of the attempted solution of an engineering problem are then incorrect.

The transposition of formulae is a skill whose lack is frequently commented on. To many academics this skill is too often taken for granted, but many students have at best a shaky grasp of what they

are attempting to do and of the 'rules' that they are using. Once again, it is practice, which is required, and examples from their engineering studies should be used as much as possible so that they see the relevance of what they are doing.

The last point applies quite generally. Although the use of 'abstract' examples has merit, every set of examples should, where possible, include at least one framed in the context of the engineering discipline relevant to each student. To make R the subject of the formula $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ is surely more interesting to an electrical engineering undergraduate than being asked to make z the subject of the formula $\dfrac{1}{z} = \dfrac{1}{x} + \dfrac{1}{y}$.

Workbook 2: Basic Functions

Functions are the verbs of mathematics and a firm grasp of their basic definitions and properties are crucial to later work. Many students are put off by the use of the notation $f(x) = x^3$ and this effect can be reduced by a gentle, notation-free introduction.

This Workbook covers the areas: the concept of a function, the graph of a function, functions in parametric form, classifying functions, linear functions, simple standard engineering functions, and the circle.

For students not versed in the idea of a function the box diagram (input - rule - output) has proved to be the most useful in discussing the topic, particularly with regard to the inverse of compound functions. By treating a few very simple functions in this way, it is then time to introduce the concept of a function as a mapping and the notation $f(x)$. Then compound functions can be studied and the box diagram approach helps to clarify the domain and range.

When presenting the representation of a function as a graph it is important to stress the word 'representation'; too many students think that the graph is the function. Now is the time to present a function as a box diagram, a rule and as represented by a graph in order to show the link between the three approaches.

At this early stage it is possible to give examples of functions described in parametric form and demonstrate how to calculate points on their graph. It is then the time to present an example of a function which would be very difficult, or impossible, to present in the standard $f(x)$ form. Incidentally, it is a good idea to now consider functions whose input variable is not x, say t .

The inverse of a function, its domain and range can be discussed first by box diagram, then by rule and then by graph, in that order. The use of the terms 'one-to-one' etc can now be safely mentioned provided that the treatment is gentle. The distinction between functions whose inverse is itself a function and those for which this is not true can be illustrated by examples such as $f(x) = x + 2$, $f(x) = 2x$, $f(x) = x^3$ and $f(x) = x^2$. Other examples can be presented via graphs.

With simple examples, and a liberal use of the graphical approach, the concepts of continuity, piece-wise definition, periodicity, oddness and evenness can extend the portfolio of functions considered. Illustrating these concepts, however crudely, by examples from engineering is particularly important here, to prevent the presentation drifting into an apparently aimless academic exercise.

When students met the topic of straight lines at school it was almost certainly not as the graphical representation of a linear function. The advantage, for example, of calculating the exact distance between two points as opposed to a graphical estimation should be stressed, by asking students to discuss how they would find the distance between two points in three dimensions and why the problem is different from the two-dimensional case.

A circle in standard form is an example of an implicit function, but that may be too deep to discuss at this level. Rather it provides the opportunity to carry out some simple coordinate geometry where the visual checks are easy to perform. The use of some of these results in the relevant engineering discipline must be demonstrated.

Polynomial functions are well behaved and can act as a vehicle for looking at concepts of oddness and evenness, decreasing, increasing and stationary. Simple rational functions, including $f(x) = x^{-1}$ allow ideas of discontinuity and asymptotes to be introduced. The modulus function and the unit step function allow students to widen their horizons on the meaning of 'function' and can be (and must be) shown to have practical engineering applications.

Workbook 3: Equations, Inequalities and Partial Fractions

The solution of simple equations underpins much of engineering mathematics. The need to know whether the equation concerned has any solutions, and if so how many is essential to the satisfactory solution of problems. Much less well understood are inequalities and a thorough grounding is required at this level. The concept of the partial fractions of a rational function is useful in so many areas, for example inverting a Laplace transform.

The areas that this Workbook deals with are: the solution of linear equations, the 'solution' of quadratic equations and polynomial equations in general, simultaneous equations, simple inequalities and their solution, determination of partial fractions.

There are examples a-plenty of linear equations in engineering and the opportunity should be taken to introduce the topic via one from the students' discipline. Linking the solution to the graph of a straight line crossing an axis is recommended. Are there any cases therefore where no solution exists? Is the solution unique and why is it important to know this?

The solution of quadratic equations provides an opportunity to bring together several areas of mathematics. Looking at graphs of various quadratics suggests the possibility of two distinct solutions, a unique solution or no solutions; algebra can verify the suggestion. The usefulness of factorisation in determining the crossing points of the graph on the horizontal axis, and the location of the stationary point of the function can be stressed, but if no factors exist students may find the method wasteful of time; honesty is essential. Completing the square is clumsier in determining two crossing points but readily yields information about the stationary point when there are no solutions of the associated quadratic equation. The formula method then comes into its own as providing relevant information in all cases; this is the time to introduce it, but do stress the relevant strengths of all the approaches.

The introduction to higher degree polynomial equations should be gentle and, in the main, concentrating on general principles. It is a good moment in the course to emphasise how the complexity of the problem increases as the degree of the polynomial increases.

The solution of simultaneous equations allows the opportunity to show the interaction between graphical indication and algebraic determination. Start with the four graphical cases of single intersection (at almost right-angles), parallel lines, coincident lines, and almost-coincident lines. Then show how the graphs relate to the algebraic 'solution'. A simple introduction to ill-conditioned equations can be given here.

The relationship between a simple inequality and points on the real line helps students to visualise the meaning of the inequality; its solution algebraically can be related to the picture. The case of a quadratic inequality is only sensibly dealt with at this level by extensive use of a graphical argument.

The first point to emphasise with expressing a rational function as a sum of partial fractions is the need to choose suitably-shaped partial fractions; the process of finding the coefficients in the numerators can then, in principle, follow successfully. It is recommended that a single strategy is employed: combine the proposed partial fractions into a single fraction with the same denominator as the original fraction; substitute suitable values of the variable into the numerators to obtain, one by one, the unknown coefficients; if this does not determine all of them resort to comparing coefficients of powers of the variable. It is suggested that at least one example is given where one of the unknown coefficients is zero to warn the students that this can happen with a general strategy and ask whether they could have foreseen that zero value.

Workbook 4: Trigonometry

There are some colleagues who believe that the sine, cosine and tangent functions should be introduced via right-angled triangles and then extended to their application to oscillatory motion. Others believe that the application to oscillatory motion should precede their link to triangles. This Workbook follows the first approach, but with suitable guidance students following the second approach can also use it successfully.

This Workbook deals with the areas of right-angled triangles and their solution, trigonometric functions of any angle, including their graphs, simple trigonometric identities, the solution of triangles without a right angle, the application of trigonometry to the study of wave motion.

In this age of the pocket calculator many students are not familiar with the exact values of the trigonometric ratios for angles such as $30°, 45°, 60°$. Does it matter? The view taken here is that it does. It is also important to note that the inverse trigonometric functions found by pressing calculator buttons may not provide the answer required. It is worth emphasising that checking whether a triangle is right-angled before solving it via sine and cosine rules can save much effort.

These days it might be argued that there is little value in knowing, or using, throwbacks such as the CAST rule; on the contrary, we argue that such knowledge and usage leads to a greater 'feel' for the trigonometric functions than mere blind button-pressing. The ability to sketch graphs of the basic trigonometric functions adds to their safe application to problem solving.

How far one should delve into trigonometric identities is a matter for debate but surely everyone should know the identity $\sin^2 \theta + \cos^2 \theta \equiv 1$ and its close relatives. An awareness of some of the addition formulae allows the 'double angle' identities to be seen as plausible.

A strategy for the solution of triangles in general, using angle sum, sine rule and cosine rule should

be presented. This is a good situation in which to explain the idea of adapting the strategy to different input information. The fact that the information given does not allow a triangle to be drawn illustrates the principle that 'if you can draw the triangle then you can calculate the unknown sides and angles'. The ambiguous case, presented pictorially first, gives emphasis to recognising that for an angle $< 180°$, knowing its sine does not define it uniquely (except, of course, for $90°$).

This Workbook assumes no previous knowledge of the subject and begins with a discussion of the trigonometry of the right-angled triangle. More general definitions are then given of the trigonometric (or circular) functions and the main trigonometric identities are obtained.

The sine rule and cosine rule for triangles are introduced and a careful discussion given of which is the appropriate rule to use in a given situation.

Detailed worked examples and a generous selection of problems (with answers) are provided at all stages.

As an application a full section is devoted to properties of sinusoidal waves whilst other smaller optional engineering examples utilising trigonometry (for example in the diffraction of sound, brake cables, amplitude modulation and projectile motion) are interspersed at appropriate intervals.

Workbook 5: Functions and Modelling

One of the difficulties found frequently by engineering students is the progression from the statement of a problem in words to a mathematical solution and its interpretation. In this context, a reasonable proficiency in a range of mathematical techniques is not sufficient. Students need to be able to abstract salient features from a real problem, make suitable assumptions, assign sensible symbols to the variables, state the mathematical problem (the *mathematical model*) and devise a strategy for solving it. Having solved it, the student needs to be able to interpret the result either in terms of the predicted behaviour or numerically (using consistent units). If the interpretation is not satisfactory the student needs to be able to refine the initial model.

This Workbook is intended to introduce this idea of the 'Modelling Cycle'. The emphasis is on 'choosing a function for a model'. Since Workbook 5 is early in the HELM series, it deals only with models that involve linear, quadratic and trigonometric functions. (The last section in Workbook 6 includes modelling examples that use the logarithmic and exponential functions.) An attempt has been made to use the 'modelling' format in all of the engineering examples in the HELM series.

Following a section on the use of linear functions as models we turn our attention to the use of quadratic functions. An obvious example is that of projectile motion. Given the relative simplicity of the quadratic function it is worth examining the assumptions made by using it to model such situations; how accurate are the predictions from the model and does it provide an exact model, or merely a very good approximation to reality? To avoid confusion in the case of two-dimensional projectile motion, it is worth spending time with the example of a stone thrown vertically upwards distinguishing between the graph of the vertical displacement against time and the actual motion of the stone.

Then we model oscillatory motion by means of the sine and cosine functions, reinforcing the introduction provided in Workbook 4. Finally, we take an example of the inverse square law model; this

is such a widely-applied model that it needs an early mastery.

The definition of the 'Modelling Cycle' and the approach adopted are similar to those used in a series of Open University courses (TM282 *Modelling by Mathematics* and MST121 *Using Mathematics*). These courses cite some engineering examples but, since the intention was to avoid contexts that required a lot of prior knowledge, several other contexts are used here. For example, the Open University materials for TM282 use three contextual themes: Vehicle Safety, Town Planning and Population Growth. Further illustrations of modelling are to be found in the texts for those courses.

More advanced modelling examples may be found in later HELM Workbooks (particularly 34, 47 and 48) and in the following references:

The Open University MST207 Mathematical Methods and Models,

J. Berry and K. Houston, *Mathematical Modelling*, publ. Edward Arnold, 1995,

K. Singh, *Engineering Mathematics through Applications*, Palgrave, 2003.

Workbook 6: Exponential and Logarithmic Functions

Two of the most hard-worked functions in models of engineering problems are the exponential and logarithmic functions. As with all work-horses, a thorough knowledge of their behaviour is essential to their efficient use; effort spent now will reap rewards over and over again.

After some examples of functions from the family a^2 , the exponential function is introduced by defining e as the limit of a sequence. A study of the slope of the tangent to curves of the family can be followed by stating that in the case of the exponential function that slope is equal to the value of the function at the point of contact. This then allows a generalisation to the slope being proportional to the value and hence to models of growth and decay, for example Newton's law of cooling.

The hyperbolic functions can be introduced at this point without too much fuss. Given the contrast between some of their properties and those of their trigonometric near-namesakes, it might be wise to hint at their use in parametric coordinates on a hyperbola (hence the trailing h).

The use of the logarithmic function in solving model equations that employ the exponential function reinforces the concept of an inverse function. The properties of logarithmic functions need treating with care: the law $\log_a(AB) = \log_a A + \log_a B$ is not true for A and B both negative, for example.

To an extent the material in Section 6.6 on the log-linear transformation duplicates that in earlier sections of the Workbook. However this section provides an alternative approach and illustrates its use in various modelling contexts. Indeed the emphasis in Section 6.6 is on the use of exponential and logarithm functions for modelling.

Workbook 7: Matrices

The algebra of matrices can be a dry topic unless the student sees that matrices have a vital role to play in the solution of engineering problems. Of course, one obvious application is in the solution of

simultaneous linear equations, but there are others, for example in the description of communication networks, and at least one such example should be introduced at the outset.

Of necessity, some groundwork must be carried out and this can be tedious; where possible, emphasise the 'natural' definitions, for example addition of two matrices of the same shape; then ask how they would add together say

$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$. In new territory nothing can be taken for granted.

Matrix multiplication is a strange process. The multiplication of two-by-two matrices can be illustrated as representing two successive rotations (use simple angles). The generalisation to three-by-three matrices is a hard one, so it is important to emphasise the scalar product basis of deriving the two-by-two entries e.g. for the product $C = AB$, $c_{11} = a_{11}b_{11} + a_{12}b_{21}$ etc. And, of course, the shock to the system of non-commutativity and results such as $AB = 0$ does not necessarily imply that either $A = 0$ or $B = 0$ (or both) should be exploited. Again it is important to give simple examples of where multiplication is not possible then give them back their confidence by giving them simple guidelines on how to negotiate the minefield.

A simple introduction to the concept of the determinant of a square matrix and some simple properties of determinants should be accompanied by an illustration of how tedious it can be to evaluate a determinant with even four rows. Please emphasise that there is a different notation for determinants than that for matrices and that they are not interchangeable.

The inverse of a two-by-two matrix is best introduced by the idea of reversing a rotation. The fact that even some square matrices do not have an inverse must be justified. That we say "*the* inverse matrix, when one exists," is a useful vehicle to explain the vital importance of existence and uniqueness theorems. Emphasise that division of two matrices is NOT an operation. Build up confidence by finding inverses of two-by-two matrices before embarking on the three-by-three examples; these might be postponed until simultaneous linear equations are introduced. (Workbook 9). The contrast between the Gauss elimination and the determinant method should be drawn. Whilst students should be made aware of both methods they should be allowed to use the method which they prefer.

Workbook 8: Matrix Solution of Equations

This Workbook introduces the methods of solution by Cramer's rule, by inverse matrices and by Gauss elimination. The relative strengths of these methods should be highlighted, in particular how they cope with systems of equations that have no unique solution.

Cramer's rule provides a simple approach to understanding the nature of solution of two equations in two unknowns and is easily extended to three equations in three unknowns. It should be noted how much more arithmetic is involved and that extension to larger systems is really impracticable.

The use of the inverse matrix provides an alternative approach, which has useful links to the underlying theory but is not much help in the cases where no unique solution exists.

In contrast, the method of Gauss elimination can, properly handled, lead to useful information when

there is no unique solution, especially if there are an infinite number of solutions. These days, students are not fluent in handling arithmetic fractions and care must be taken to avoid examples, which lead to anything but the simplest fractions. The method provides a variety of routes to the solution and this fact may cause students to be wary of using it. A warning could be usefully issued by showing the exact solution to a system of ill-conditioned equation alongside a Gauss 'solution'.

Workbook 9: Vectors

In this Workbook we provide a careful introductory account of vectors together with a few basic applications in science and engineering. The approach is relatively conventional, starting with the elementary ideas, approached graphically, of vector addition and subtraction and the multiplication of a vector by a scalar. Applications involving the addition and resolution of forces and a discussion of the forces on an aeroplane in steady flight are provided. (Some elementary knowledge of trigonometry is required here and later in the Workbook.) An opportunity can be taken to discuss the contrast between displacement of an object (vector) and the distance it travels (scalar).

We underline vectors, which better reflects how students and lecturers write them (rather than emboldening them as is done in textbooks). Note that we write $d\underline{r}/dt$ (underlining r only) but $\underline{dr}$ (underlining the expression dr).

Cartesian representation of vectors, firstly in two and then in three dimensions is covered including position vectors. The basic unit vectors are denoted by $\underline{i}, \underline{j}$ and $\underline{k}$. It is important not to rush through the two-dimensional case: once this is well understood the three-dimensional case can be sold as a straightforward extension (and the reason for 'the adding of an extra term' allowing us to move into three dimensions can be explained).

A careful coverage of the scalar (or dot) product of two vectors and its properties, together with some applications in basic electrostatics, is followed by the corresponding vector (or cross) product. Carefully explain why $\underline{a} \times \underline{b} = -\underline{b} \times \underline{a}$; it is a result to frighten the faint-hearted. In the latter the determinant form for evaluation in Cartesian coordinates is used and simple applications to the torque of a force are discussed. Triple products are covered only in exercises. Many students find difficulty with problems in three-dimensional statics in their engineering modules and the use of the vector product in tackling these problems can be a good selling-point here (case study or coursework, perhaps).

The final section involves some geometrical aspects of vectors particularly direction cosines and direction ratios and vector equations of lines and planes. The simplicity of approach using vectors can be highlighted by contrasting it with the alternative approach.

Workbook 10: Complex Numbers

Complex numbers are a basic tool in engineering and science and this Workbook gives a gentle introduction to their properties and uses. A non-rigorous approach is adopted with a complex number being denoted by $z = a + ib$ in Cartesian coordinates and by $z = r(\cos\theta + i\sin\theta)$ in polar coordinates. The complex exponential form is also covered.

The Argand diagram provides a useful means of visualisation of complex numbers but care must be taken to stress that a point on the diagram *represents* a complex number and is not the number itself. The same point can be made when representing a complex number as a vector.

The Workbook initially covers the algebra of complex numbers in Cartesian form and glances briefly at the solution of polynomial equations with real coefficients. Given the poor grasp of real number algebra among many students, it is wise not too assume too much algebraic fluency, so take it slowly. Assuming some knowledge of trigonometric identities allows the introduction of the polar form and its advantage over the Cartesian form when *carrying* out multiplication and division (and taking powers). A simple application to rotations about a point provides a gentle illustration.

Use of the exponential form leads to a discussion of the relations between trigonometric and hyperbolic functions and identities and the Workbook concludes with De Moivre's theorem for finding powers and roots of complex numbers. It is worth remarking that the nth roots of a given complex number share the same modulus and therefore lie on a circle in the Argand diagram and are spaced $\left(\frac{360}{n}\right)^\circ$ apart along the circumference.

Workbook 11: Differentiation

At the outset it should be remarked that a sensible ordering of the sections in Workbooks 11 and 12 enhances the presentation of the material to students. After a new technique a reinforcing application example aids the learning process. There was a time when virtually all students embarking on an engineering degree course were well versed in the rudiments of differentiation; that is no longer the case. When the topic was covered in secondary education much time was devoted to it; it is not a topic that can be skimmed over.

Many of today's students seem to have difficulty understanding the concept of rate of change and this needs careful attention. The average rate of change of a function over a given interval should be introduced and the increment in the independent variable denoted by h: the use of δx can come later - to make plausible the notation $\dfrac{dy}{dx}$. Linking the instantaneous rate of change to the gradient of the tangent should be done at the same time.

It is important to define carefully the terms 'differentiable', derived function' and 'derivative' and to distinguish clearly between them. It is also straightforward to give examples where a function is not differentiable at a point: the modulus function, ramp function and unit step function are good examples.

Familiarisation with the table of basic derivatives (derived functions) should be encouraged and the results interpreted on graphs of the appropriate functions. Do stress that angles in this context MUST be measured in radians.

A graphical explanation of how the derivatives of $f(kx), kf(x), f(x+k), f(x)+k$ are related to that of $f(x)$ has been found to help. Should it be necessary to explain why the derivative of a constant function is zero? Probably it is.

The rule for differentiating a linear combination of two functions (and the special cases of sum, difference and scalar multiple) is reasonably intuitive and need not be overstated. Confidence is the

keyword here.

The second derivative could be presented in terms of displacement, velocity and acceleration - and a good time to explain that x is not the only notation for an independent variable. The notation $\dfrac{d^2y}{dx^2}$ etc. does seem clumsy and needs justification.

The remaining sections of the Workbook deal with the techniques of differentiating products and quotients, the chain rule and parametric and implicit differentiation. The examples chosen must be simple illustrations; don't lose the wood of the technique in the trees of awkward differentiation. A useful approach is to illustrate some applications of one technique before moving onto more advanced techniques; in particular the last three techniques and especially the last two can be postponed until earlier techniques have been mastered and time allowed for their absorption.

Workbook 12: Applications of Differentiation

The application of differentiation to the finding of the tangent and (indirectly) the normal to a curve should be straightforward but it will help if the results are interpreted on a sketch graph of the relevant function. This suggests the use of simple functions until the student is confident in using the strategy.

Stationary Points and Points of Inflection

Most students have an imperfect understanding of the definitions of local maximum, local minimum and point of inflection. Simple graphs can be used to illustrate these features.

Of the following three statements only the first two are known with any certainty by most students:

Given a twice differentiable function f for which $f'(a) = 0$

(1)   If $f''(a) > 0$, then $f(x)$ has a minimum when $x = a$,

(2)   If $f''(a) < 0$, then $f(x)$ has a maximum when $x = a$,

(3)   If $f''(a) = 0$, then $f(x)$ has minimum or a maximum or a point of inflection when $x = a$.

Many students think (3) **always** leads to a point of inflection but the graph of $f(x) = x^4$ clearly shows this to be untrue when $x = 0$.

Another misconception is that a point of inflection **requires** $f'(a) = 0$. This is not true as can easily be seen, for example, on the sine curve. This raises another point - for any continuous function there is always a point of inflection between every local minimum and local maximum. The graph below highlights these features.

$$y = x^4 - 4x^3 - 2x^2 + 12x + 2$$

Maxima and Minima without Calculus

Students all too readily turn to the calculus when needing to find maxima and minima. There are, however, cases when alternative approaches are simpler, quicker or more informative:

Example 1

Find the minimum value of $f(x) \equiv x^2 + 2x + 3$.

Completing the square gives $f(x) = (x + 1)^2 + 2$.

This clearly is a minimum when $x = -1$ and there $f(x)$ has value 2.

Example 2

Find the maximum value of $f(x) \equiv 2\sin(x) + 3\cos(x)$.

Using the trigonometric identity $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$ and utilising the triangle in the diagram we have

$$f(x) = \sqrt{13}\left[\frac{2}{\sqrt{13}}\sin x + \frac{3}{\sqrt{13}}\cos x\right]$$

$$= \sqrt{13}[\cos\alpha\sin x + \sin\alpha\cos x]$$

$$= \sqrt{13}\sin(x+\alpha)$$

This clearly has a maximum value of $\sqrt{13}$ at $x = \dfrac{\pi}{2} - \alpha$ (for example).

One point of caution should be stated. When applying differentiation to find for example the dimensions of a rectangle of fixed perimeter and maximum area students often get confused between the graph of the function being maximised and the geometry of the problem. It is also useful to remember that differentiation can find a local maximum and physical considerations can be brought in to justify the term "absolute" maximum. Also, in some physical problems, with the knowledge of the behaviour of the function at the physical extremes of the independent variable the nature of the single stationary point can be decided without the need to differentiate a second time, or use the first derivative test.

The Newton-Raphson method is straightforward to derive and relatively easy to apply. Mention should be made of its proneness to failure under certain circumstances. The presentation of curvature should be illustrated by simple examples so that the student can relate algebraic results to the graph of each function. The differentiation of vectors in Cartesian component form follows an intuitive path and this should not be clouded by too much rigour.

Workbook 13: Integration

The concept of an indefinite integral is introduced in the traditional way, as the reverse of differentiation. It is helpful to illustrate this by integrating the velocity function to obtain the displacement function and noting that we cannot say where an object is unless we knew where it was at an earlier time - hence the need for the arbitrary constant. Using the table of derivatives 'back to front' and the rule for linear combinations of functions, which parallels that for derivatives ought not to present many difficulties.

The extension to definite integrals by itself is not too abstruse but the notation in the statement $\displaystyle\int_{1}^{2} 3x^2\, dx = \left[x^3\right]_{1}^{2} = 8 - 1 = 7$ is quite deep and needs a gentle presentation.

Surely one of the great moments in learning mathematics is to realise that the area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ can be found exactly by evaluating

the definite integral $\int_a^b f(x)dx$. Make the most of that moment. Simple examples, please. The area bounded by the sine curve over a quarter-period has a satisfyingly simple value.

Further techniques can be postponed until some further simple applications such as those in Workbook 14 have been looked at.

The techniques of integration by parts, by substitution and by partial fractions are best treated in two stages; first use really simple examples in order to allow students to gain confidence in the basics of each technique, preferably with illustrative examples from engineering to demonstrate that their use is not just 'pure mathematics', then revisit with more advanced versions of the techniques.

As a final phase, integration of functions involving more complicated trigonometric expressions can be tackled.

Workbooks 14 & 15: Applications of Integration 1 & 2

These two Workbooks present a selection of the most common applications of definite integration. Workbook 14 first deals with definite integration as the limit of a sum, then considers mean value and root-mean-square value of a function, volumes of revolution and the lengths of arcs and the area of surfaces of revolution. Workbook 15 considers the integration of vectors, finding centres of mass, and finding moments of inertia.

The importance of recognising a definite integral as the limit of a sum cannot be over-emphasised: it allows the correct establishment of a definite integral whose value is that of the physical quantity concerned. The remainder of the solution is the use of techniques of integration met in Workbook 13. The first examples of each application should lead to very simple integrands. The results should always be examined to see whether they are sensible in the light of the problem.

The mean value of the sine function over a quarter-period, over a half-period and over a full period can be compared and the need for a more meaningful measure of the 'average' of the sine function suggested. Of course, the root-mean-square value is more tricky to evaluate and the double angle formula should be provided as a tool to be picked up and used.

When finding volumes of solids of revolution the cylinder, cone and sphere are the obvious examples and the placing of an axis of symmetry along the x-axis should be highlighted. Careful explanation of the division of the solid into the union of a set of non-overlapping discs should be followed by remarking how an approximating sum can be transformed into a definite integral yielding the exact answer - one of the glories of the calculus.

Approach the problem of finding the length of arc by first using a line parallel to the x-axis, then a line inclined to the x-axis and thus make plausible the length of a general arc. The resulting formula can then be shown to yield the same results as in the first two cases. The formulae for this application and that of surface area only yield simple integrations in a few cases and it is worthwhile pointing out that other examples may require much more complicated methods of integration, or perhaps need to employ numerical approximate methods.

The integration of vectors in Cartesian coordinate form is relatively straightforward and can be dealt

with immediately after differentiating them - use velocity goes to displacement as a suitable vehicle for discussion. When discussing the centre of mass of a plane uniform lamina the use of symmetry is a vital ingredient; sensible positioning of the axes may simplify the resulting integrations, not least finding the value of the area itself. Many students do not have a grasp of the idea of a moment and a matchstick model is not too elementary a teaching aid. Finally, the calculation of moment of inertia is not a great leap mathematically but does need a careful foundation of the physical concept. Many students have only a rudimentary knowledge of mechanical principles and very little can be assumed.

Workbook 16: Sequences and Series

This Workbook gives a comprehensive introduction (adequate for most science and engineering students) of these topics. Apart from basic algebra there are no significant prerequisites. Even sigma notation is introduced as a new topic.

In the first section the concept of sequences and their convergence or otherwise is introduced. Arithmetic and geometrical sequences (progressions) are covered and formulae obtained for the sum of n terms in each case.

We then move on to more general infinite series. The meaning of convergence is carefully explained in terms of partial sums. The ratio test for the convergence of series of positive terms is explained together with a brief mention of other convergence tests and the ratio test is also used to briefly consider conditional or absolute convergence for more general series.

A brief section follows on the binomial series and the binomial theorem for positive integers. This is followed by a section on the convergence of power series in which the idea of the radius of convergence is carefully explained. Various properties of convergent power series are also discussed such as their differentiation and integration.

The most useful aspect of convergent power series for applications (their ability to represent functions) is the subject of the final section in which both Maclaurin series and, briefly, Taylor series are discussed.

Workbook 17: Conics and Polar Co-ordinates

This Workbook offers a brief but, for most purposes, adequate introduction to the Co-ordinate Geometry of the three conic sections - the ellipse, parabola and hyperbola - with the circle also being introduced as a special case of the ellipse. The basic Cartesian equations of the conics with axes parallel to one or other of the co-ordinate axes are obtained in the first section and mention made of more general cases.

In the second section polar co-ordinates are introduced and simple curves whose equations are given in polars are described while the equations of the standard conics in polars are also explained.

Finally parametric descriptions of curves are covered including those for the standard conic sections.

The prerequisites for this Workbook are relatively modest, with some basic algebra including completion of the square and algebraic fractions and knowledge of trigonometric functions being required

for the first two sections. Knowledge of hyperbolic functions and simple differentiation is required for the final section but detailed discussion of parametric differentiation is covered in Workbook 12.

The Workbook contains a good number of fully worked examples and many exercises for all of which answers are provided.

Notes relating to Workbook 17 page 20:

The general conic

Although every conic is of the form

$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$ (with not all A, B, C zero),

not every equation of that form represents a conic. (The equation $x^2 + 1 = 0$ is an obvious counter-example!)

Meaningful cases are:

Ellipse: $AC - B^2 > 0$ [eccentricity $e < 1$]

  Possibilities: Ellipse, Circle, Single point.

Parabola: $AC - B^2 = 0$ [eccentricity $e = 1$]

  Possibilities: Parabola, Two parallel lines, Two coincident lines.

Hyperbola: $AC - B^2 < 0$ [eccentricity $e > 1$]

  Possibilities: Hyperbola, Rectangular hyperbola, Two intersecting lines.

Definition of conic

A conic is the locus of a point, which moves in a plane so that its distance from a fixed point called the **focus** bears a constant ratio called the **eccentricity** to its distance from a fixed straight line in the plane called the **directrix**.

If (p, q) are the coordinates of the focus, e is the eccentricity, and $ax + by + c = 0$ is the equation of the directrix then the equation of the conic is

$$(x - p)^2 + (y - q)^2 = e^2(ax + by + c)^2/(a^2 + b^2)$$

which can be rearranged into a biquadratic of the form

$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$ (as stated at the beginning) where

$$
\begin{aligned}
A &= a^2 + b^2 - a^2 e^2 \\
B &= -abe^2 \\
C &= a^2 + b^2 - b^2 e^2 \\
D &= -p(a^2 + b^2) - ace^2 \\
E &= -q(a^2 + b^2) - bce^2 \\
F &= (a^2 + b^2)(p^2 + q^2) - c^2 e^2
\end{aligned}
$$

Workbook 18: Functions of Several Variables

This Workbook is in four sections. We introduce first of all functions of several variables and then discuss graphical representation (surfaces) for the case of two independent variables x and y.

The heart of the text is the section on partial differentiation (for which, naturally) a basic knowledge of differentiation in the case of one independent variable is needed. The material is actually quite straightforward and proceeds only to second partial derivatives including mixed derivatives.

The third section covers stationary points for functions of two variables with computer drawn diagrams to illustrate the various possibilities. The location of stationary points using partial differentiation is carefully discussed and the second derivative tests to determine the nature of these points clearly set out but not proved.

The final section covers errors and percentage changes using partial differentiation. Many worked examples including applications are provided at appropriate intervals.

Note relating to Workbook 18 page 18:

It is difficult to give a watertight yet straightforward explanation of the tangent surface in 3-D although the concept is readily understandable. We have therefore not attempted to be rigorous here.

Workbook 19: Differential Equations

This topic is one of the most useful areas of mathematics for the scientist and engineer in view of the extensive occurrence of differential equations as mathematical models. This Workbook provides a sound introduction to first and second order ordinary differential equations (ODEs). No previous knowledge of the topic is required but a basic competence at differentiation and integration is needed.

The first section introduces ODEs through a model and then discusses the basic concepts of order, type and general solution together with the need for additional condition(s) to obtain a unique solution.

The section on first order ODEs covers separation of variables and the conversion of linear ODEs into exact equations of the form

$$\frac{d}{dx}(y \times f(x)) = g(x)$$

via the use of an integrating factor.

As would be expected in an introductory treatment, second order ODEs are restricted to the linear constant coefficient type but these of course have many applications. Both homogeneous and non-homogeneous types are fully discussed. Knowledge of complex numbers is useful here for dealing with forcing functions involving sinusoids. A number of engineering application examples from electric circuit theory are fully worked out.

The final consolidating section of the Workbook returns to the applications aspect with, among other topics, a reasonably full discussion of mechanical oscillations.

Workbook 20: Laplace Transforms

This Workbook is in many ways a counterpart to Workbook 21 on z-transforms but the two books are independent. Workbook 20 contains a thorough introduction to the Laplace Transform and its main properties together with various applications. Only the one-sided Laplace Transform is considered. A facility with integration is a valuable prerequisite.

The first section introduces causal functions of the form

$$h(t) = f(t)u(t)$$

where $u(t)$ is the unit step function. Simple properties of causal functions are studied as are delayed functions of the form

$$f(t-a)u(t-a)$$

and signals of finite duration of the form

$$f(t)[u(t-a) - u(t-b)]$$

The definition of the Laplace Transform is used to transform various common functions including delayed functions. Inversion of transforms is demonstrated using partial fractions (an outline knowledge of which is assumed). Introduction of the shift theorems both in t and the Laplace variable s allows a wider range of transforms and inversions to be dealt with.

The Laplace Transform of derivatives is covered and this leads on naturally to the solution of ordinary differential equations and systems of these. Specific electrical and mechanical engineering examples are demonstrated. A section on convolution integrals and the time convolution theorem is provided and the concept of transfer or system function for linear systems follows naturally. The negative

feedback system is investigated. These application topics enhance the value of the Workbook in a specifically engineering context.

Workbook 21: Z-Transforms

This topic does not appear in older classical texts covering mathematical methods in science and engineering and hence the detailed coverage from first principles provided in this Workbook should be of considerable value particularly to those involved in discrete signal processing and digital control.

The Workbook begins with a basic discussion of sequences and simple difference equations before moving on to a full discussion of the z-transform and derivations of the transforms of important sequences. The transform is introduced ab initio with no previous knowledge of Laplace transforms required.

The shift properties of the z-transform are fully discussed and applied to the solution of linear constant coefficient difference equations. Inversion of z-transforms by partial fractions and by the use of residues is covered although no detailed knowledge of Complex Variable Theory is needed. A detailed application of the solution of difference equations to obtain currents in a ladder network provides a good consolidating example.

A full section is devoted to the concept of transfer (or system) function of discrete systems and the time convolution theorem is also discussed in some detail.

A final (optional) section briefly extends the theory to sampled functions and the relation between the z and Laplace transforms is finally obtained.

Workbook 22: Eigenvalues and Eigenvectors

This Workbook assumes a knowledge of basic matrix algebra and an outline knowledge of determinants although brief revision notes are provided on the latter topic. After a brief rsum of the various situations that can arise in solving systems of linear equations of the form

$$CX = K$$

a full discussion is given of the eigenvalue problem

$$AX = \lambda X$$

and calculation of the eigenvalues λ and the eigenvectors X is demonstrated both using the characteristic equation and later by numerical methods.

A detailed section on applications considers diagonalisation of matrices with distinct eigenvalues and this leads on to solving systems of differential equations (such as coupled spring systems) by the 'decoupling' method.

Further applicable theory deals with the pleasant properties of symmetric matrices in this context and an outline is given of the situations that can arise with general matrices possessing repeated eigenvalues.

A student who has mastered the material of this Workbook is in a strong position to study specific applications such as advanced dynamics and modern control.

Workbook 23: Fourier Series

Fourier series are a well-known tool for analysing engineering and physical systems involving periodic functions (signals). They also arise in solving partial differential equations analytically by the separation of variables method. The basic idea of course is to represent a periodic signal in terms of sinusoids and co-sinusoids. Full details as to how to do this are given in a plausible rather than rigorous fashion in this Workbook. Facility with integration is the main prerequisite of the reader.

An introductory section outlines the basic jargon of the topic (frequency, harmonics etc) and then, after a discussion of the relevant orthogonality properties, we show how to obtain Fourier series for functions of period 2π and then of more general period.

The particular form of the Fourier series for functions which are odd or are even is covered and this leads on, after a brief discussion of convergence, to obtaining Fourier sine or cosine series for functions defined over a limited interval.

The final section of the Workbook gives a detailed coverage of the complex exponential form of Fourier series together with Parseval's theorem and a brief discussion of electrical engineering applications.

Fourier transforms are covered separately in Workbook 24, and the use of Fourier series in connection with partial differential equations is introduced in Workbook 25.

Workbook 24: Fourier Transforms

In this Workbook a brief but adequate introductory account is given of the complex exponential form of the Fourier transform. An informal derivation of the transform as a limiting case of Fourier series is given and mention made of the different ways of writing the transform and its inverse. Our choice is to put the factor $\dfrac{1}{2\pi}$ in the formula for the inverse and to use the angular frequency ω in the complex exponential factors.

The Fourier transforms of common signals such as exponential and rectangular are derived from the definition after which the main properties of the transform are obtained. The time and frequency shift (or translation) properties, the frequency and time differentiation properties and the form of the transform for even and for odd functions are among the topics discussed here.

Brief plausible discussions of the Fourier transforms of the unit impulse function (also known as the Dirac delta function) and of the Heaviside unit step function are given in the final section.

A generous selection of worked examples and of exercises (with outline answers) is provided. There

is little specific discussion of applications of Fourier transforms but a student who has mastered the contents of the Workbook would be in a good position to use them in specific fields.

Workbook 25: Partial Differential Equations

This topic is of course a very broad one. This Workbook is restricted to consideration of a few special PDEs but they are ones which model a wide range of applied problems. No previous knowledge of PDEs is required but a basic knowledge of partial differentiation and the ability to solve constant coefficient ordinary differential equations is assumed although brief revision of the latter topic is given. Numerical methods of solution are not dealt with.

The main PDEs studied - the two-dimensional Laplace equation, the one-dimensional wave equation and the one-dimensional diffusion (or heat conduction) equation - are fully discussed (although not derived in detail) so that the student knows how the detailed mathematics to follow may be applied. Various types of initial and boundary conditions are mentioned.

The separation of variables method of solution for PDEs is the core of the Workbook and this is developed carefully, initially looking at problems where Fourier Series do not arise. Emphasis is given to the fact that the method is a logical sequence of steps relevant to a wide range of problems. The Workbook concludes with problems where Fourier Series are required, the latter being the subject of Workbook 23 in this series.

Workbook 26: Functions of a Complex Variable

This Workbook builds especially on the work on complex numbers covered in Workbook 10. The six areas that this Workbook deals with are: the basic ideas of a function of a complex variable and its derivatives, the Cauchy-Riemann equations, the standard complex functions, basic complex integration, Cauchy's theorem, and singularities and residues.

Starting with some simple complex functions and their evaluation, the idea of the limit of a function as the input variable tends to a special value is best tackled by choosing examples where the limit is path-dependent, followed by one where it is not; in the latter case the fact that the limit does not depend on the two or three paths chosen does not imply *per se* that the limit is path-independent, and raises the question as to how to be sure that it *is* path-independent. The derivative of a complex function should follow, with particular emphasis being placed on the idea of a singular point and on the similarity between processes of differentiating a complex function and a real function at a regular point.

The use of the Cauchy-Riemann equations to answer the question of analyticity of a function should be a demonstration of the elegance of this area of mathematics, and leads nicely into the idea of conjugate harmonic functions. The explanation of the role of these functions in an application of Laplace's equation should be taken. The usefulness of a mapping being conformal will have more impact if related to an example from the appropriate engineering discipline without the need to attempt the solution of that example. With what could be a relatively abstract area of mathematics, the aim should be first to show relevance, in the hope of producing sufficient interest to go through the necessary algebra.

When introducing the standard functions of a complex variable, the similarities to and differences from the real counterparts need careful treatment. The case of multiple solutions to the equation $e^z = 1$ contrasts nicely with the unique solution to the equation $e^x = 1$.

Section 4 gives the reader a first glance at complex integration. The message is 'keep the integrands simple'. Again, it is useful to give an example where the integral is path-dependent and one where it appears to be path-independent, and raise the question of how we could verify the path-independence in any particular case. This area of mathematics is awash with elegant theorems and may appear daunting to students not weaned on theorems at school; a gentle approach is recommended.

The beauty of the simplicity of Cauchy's theorem and its wide applicability has the potential to excite the imagination of even weak students, if sensitively presented. A gentle gradation of examples is especially called for here. The extension to Cauchy's Integral Formula should follow immediately to link the two results at the earliest opportunity.

The classification of singularities of a complex function is a natural consequence of its Laurent series, which in turn is a neat extension of its Taylor series. Experience shows that Taylor series can be particularly daunting and the use of simple functions and a relatively low expectation of the depth of coverage at a first run-through is advisable.

The residue theorem can be introduced by means of simple examples and forms a suitable point at which to conclude the first venture into functions of a complex variable.

The aim throughout should be to enthuse the student by the elegance and simplicity of the topic and not go too deeply into each sub-topic. Harder examples are best tackled when the imagination has been fired and this is done by keeping the examples simple; far better not to tackle harder examples at this stage than to turn the students off.

Workbook 27: Multiple Integration

Multiple Integration is a topic following on from definite integration and is often taught in the second year of University courses. The approach taken here has been moulded by modules taught to Mechanical and Aerospace Engineers in recent years.

The comparison with "single" integration (i.e. volume under a surface plays an analagous role to area under a curve) is quite deliberate as are the use of the terms "inner" and "outer" integrals and the demonstration of double integration over a rectangular region (27.1) before going onto non-rectangular regions (27.2). The majority of the examples given in a rectangular geometry are in terms of variables x and y, reflecting the fact that most of the applications given involve integration over an area or volume. However, at least one example (in 27.1) draws attention to the use of alternative notation in terms of s and t.

Similarly, most examples in a circular geometry make use of r (or ρ) and θ (or in spherical geometry, r, θ and ϕ). However, the attention drawn to s and t in 27.1 reinforces to the student that there is nothing "sacred" about the variable names used. Similarly, while the notation dA or dV is used to represent an area element or a volume element, it should be appreciated that other notation could be used. While 27.4 is concerned with the Jacobian and change of variables, it was felt the transformation to plane polar coordinates was sufficiently universal to allow this special case in 27.2

and let 27.4 show this case in context and look at other possible transformations.

Note relating to Workbook 27 page 64:

Note that here in discussing Jacobians the notation of two vertical bars is used in close proximity for two quite different concepts: modulus and determinant.

Workbook 28: Differential Vector Calculus

The main thrust of this Workbook is in 28.2. The introduction of the vector differential operators div, grad and curl with a necessary background to the subject in terms of scalar and vector fields is given in 28.1, and 28.3 developes the ideas in alternative coordinate systems. The underline notation (e.g. $\underline{u}$) has been used for vectors and the cartesian unit vectors expressed as $\underline{i}, \underline{j}$ and $\underline{k}$, giving consistency with Workbook 9.

However, two sets of notation are used for the vector differential operator i.e. the use of the words grad f, div $\underline{F}$ and curl $\underline{F}$ are sometimes used rather than the ∇ notation ($\nabla f, \nabla \cdot \underline{F}$ and $\nabla \times \underline{F}$). It was felt important to introduce students to both notational forms as the word notation often puts the mathematics in more of a physical or engineering context while the operator often simplifies calculation.

Originally, 28.3 on vector derivatives in orthogonal curvilinear coordinates was intended to be more general i.e. to mention cylindrical polar coordinates and spherical polar coordinates in the context of being special cases and to look at a few more coordinate systems and how to calculate relevant vector derivatives. However, pressure on space did not allow this and the Section was revised to include only cylindrical and spherical polar coordinate systems. [For these other cases, refer to *Handbook of Mathematical Formulas and Integrals*, Alan Jeffrey, Academic Press.]

In 28.3, unit vectors $\hat{\rho}, \hat{\phi}$ and $\hat{z}$ (cylindrical polar coordinates) and $\hat{r}, \hat{\theta}$ and $\hat{\phi}$ (spherical polar coordinates) are used. Of course, these are equivalent to $\hat{\rho}, \hat{\phi}$ and $\underline{i}, \underline{j}$ and $\underline{k}$ for Cartesian coordinates. In fact, the vectors $\hat{z}$ and $\underline{k}$ are the same vector but the notational form chosen fits the form of the other vectors under consideration i.e. $\underline{k}$ for cartesian coordinates and $\hat{z}$ for cylindrical polar coordinates.

Workbook 29: Integral Vector Calculus

This Workbook uses the same notation for vectors and vector derivatives as Workbook 28. Subsection 1 of 29.1 on Line Integrals was a candidate for being 27.5 (i.e. in the Workbook on Multiple Integrals) but it was decided to include it here in condensed form (i.e. at the beginning of the Section on Vector Line Integrals).

A possible challenge when carrying out integrals over a sloping area is of expressing dS correctly in terms of dx and dy (or dz and one of dx or dy). This point has been emphasized in the text and in a couple of examples.

The convention used was that the mathematical expressions were referred to as theorems e.g. Gauss' theorem while the term law was reserved for the application to electromagnetism (see Engineering Example).

Clearly, these topics (particularly Green's theorem) could be mentioned in greater depth.

Workbook 30: Introduction to Numerical Methods

The Workbook assumes familiarity with basic matrix theory. Although more can be said to those who know more about matrices, what is assumed is the realistic minimum.

Rounding error and Conditioning
The key here is to ensure students have an awareness that errors due to rounding (and indeed to any other source) can build up, and that injudicious choice of operations can exaggerate the error.

Gaussian elimination and partial pivoting
Solving simultaneous equations needs care if it is to be effective. Although to many Gaussian elimination is common sense (and the impressive name may cause some not to notice that this is what you might do if nobody had told you how to solve equations!) there are the issues of errors building up. Pivoting is a way to tackle this, effectively by avoiding dividing by small numbers.

LU decomposition
In principle one could solve simultaneous equations by inverting the matrices involved, and there is an explicit algorithm for finding inverses. However, while this is fine in theory, the algorithm involves many determinants and as the determinant of an $n \times n$ matrix can have $n!$ terms, this is not really practical for even modest values of n. The L-U decomposition is much simpler, and you could if you wished ask students to estimate the number of operations involved and compare it with inverting.

Matrix norms
The condition number gives an estimate of the accuracy of the solution process of a matrix equation, in that, very roughly you might expect the errors in the solution to be the size of those on the right hand side multiplied by the condition number. For square symmetric real matrices, the choice of norm which gives the smallest condition number will make it the ratio of the eigenvalues of largest and smallest modulus: this will make sense only to students who know something about eigenvalues.

Iterative methods
Jacobi and Gauss Seidel iteration: see Engineering Example 1 for a case where these take a long time to converge.

Workbook 31: Numerical Methods of Approximation

Polynomial approximations
Approximation by polynomials is common sense in that we want to approximate the unknown functions by something simple, and polynomials are simple functions. Lagrange interpolation produces an approximation that is exact at certain points. It may be worth exploring with the students just how certain an engineer is going to be about the data that are being interpolated, and to look at the effects of changes, especially if two of the x-values are close.

For students whose background is suitable one can mention approximation by other types of functions (e.g. trigonometric polynomials), or functions, which share properties which the solution is known to have. Least squares approximation of straight lines is another approach: a polynomial of degree 1 is being made to fit the data as well as possible. One could use the best least squares approximation of N points by polynomials of some chosen degree, to make the answer less susceptible to changes in the data. Exploring the effects here with a real set of data relevant to the students might be useful.

Numerical integration
Consider the size of the errors for the composite trapezium (ask the students to estimate it). In many cases this is small enough that nothing more sophisticated is needed, although there are more accurate ways of doing numerical integration: they are also more complicated.

Nonlinear equations
The bisection methods, although slow, will always work. Newton-Raphson is much more efficient, if the starting point is "close enough", but requires the derivative to be calculated. It can be shown (and Figure 11 almost does this) that Newton-Raphson will work provided the second derivative does not change sign in the area considered. There are more complicated ways of guaranteeing convergence if the derivative does change sign.

For students and teachers willing to investigate this, try Newton-Raphson for solving $x^3 - x = 0$, with various starting points. Which root does it converge to? This is quite sensitive near 0, where the second derivative does change sign.

The last word ...
Student comment: "I am amazed that you can base a whole module around guessing and still make it sound credible."

Workbook 32: Numerical Initial Value Problems

Much of this Workbook deals with PDEs and, although most Engineering students will not have encountered this, the Workbook reflects the mathematical difficulties involved in analysing PDEs. It is important that students appreciate that stability and convergence are important, even though the analysis may be hard.

General linear multistep methods
It's worth making the analogy with solving ordinary equations, that some of the apparently more awkward formulations may be easy to use as iterative methods.

Runge-Kutta method, predictor corrector methods
This gives a way of finding a reasonable initial estimate of the solution, which can be used in the corrector method: this effectively turns an implicit method into an explicit iteration. As that can be iterated, the result may be used to obtain good accuracy, provided we use something to get it started.

The sections on parabolic and hyperbolic PDEs, both of which usually arise in the form of initial-value problems, illustrate the care needed to make the methods work on particular PDEs, and that the same method will not necessarily work on other types of PDE.

Workbook 33: Numerical Boundary Value Problems

Two point boundary value problems
These link well with some of the earlier work on Workbook 30, and it is worth emphasising that the numerical schemes, which are reasonably intuitive, give rise to potentially large sets of simultaneous equations, with all the difficulties discussed in Workbook 30. The methods there, however, will allow solutions to be found.

Elliptic PDEs
These normally arise in the form of boundary-value problems, and the Workbook gives methods for Laplace's and Poisson's equations, and that it is known that these converge. The matrices involved are all "sparse" in that they have a structure and most of the entries are zero. Finding ways of exploiting the structure of the matrices can lead to improved numerical methods for their solution. Again the techniques of Workbook 30 are important, and the students should be able to see that further progress may need more attention to the methods of solving set of equations, or of matrix iteration.

Workbook 34: Modelling Motion

This Workbook relates to the modelling ideas in Workbook 5 and Workbook 6 Section 6.6. Workbook 34 would be best used to consolidate the material in Workbooks 9 to 15 (i.e. vectors and differential and integral calculus). It does so in the context of the dynamics of rigid body dynamics (projectiles, friction forces, motion on an inclined plane, motion in a circle, 3D examples of motion and resisted motion). The level is most appropriate to first year engineering mathematics. However it might form the basis for further trips around the modelling cycle in later years. Further developments might include the numerical solution of the differential equation for resisted motion that is not linear in velocity, and more realistic modelling of vehicle motion on bends.

Workbooks 35-46: Probability and Statistics

Probability and statistics deal with uncertainty and variation. Engineering students do not always easily feel comfortable with these ideas, being perhaps more at home with the apparent certainties of physical laws. In the world in which engineers work, of course, variation does exist, in materials, in the environment, in the wear and tear to which products are subjected and so on. Therefore there is also uncertainty.

Engineering students often need help in coming to terms with these concepts and with the idea that the "answer" to questions is often a statement, of one type or another, about the remaining uncertainty in a situation rather than a fixed value for some physical quantity. Physical analogy or "feel" does not always come so easily with such ideas. In students' efforts to deal with this, motivation is often of great importance and motivation is easier when the student sees the material as relevant. This is one reason why we have attempted to provide examples and explanations, which relate to engineering. It would, of course, be impractical to provide examples of every topic drawn from every specialised branch of engineering but students should be able to be less narrow than this and tutors may be able to provide additional, more specialised, examples by changing the "stories" in some of ours. The second reason for trying to use relevant engineering examples and illustrations

is to help students to attain that "feel" which will help them to grasp what is going on. This is made easier if they can relate probability statements, for example, to frequencies of events of familiar types. So, for example, it may be easier at first to think of 100 light bulbs, or whatever, of which "on average" one in twenty, that is 5, might be expected to fail before a certain time, rather than talk more abstractly of a probability of 0.05. Similarly, it might help to think of a product with two components, A and B, where one in ten products "on average" has a defect in component A and one in five products "on average" has a defect in component B, regardless of the state of component A. Then, out of 100 products, we might expect on average 'one in five of one in ten products', that is one in 50, to have defects in both components.

There is more than one school of thought in probability and statistics but presenting both of the major approaches in introductory material such as this would risk causing confusion. Although the Bayesian approach has made major advances in recent years, it is probably still true that the approach which engineering graduates will be expected to have learned is the more traditional frequentist approach. Therefore probability is presented in terms of limiting relative frequency and statistical inference is presented in terms of the familiar ideas of estimators, confidence intervals and significance tests. Thus properties can be explained in terms of long-run frequencies. This is not too difficult in the context of a mass-production industry, for example, although care is still required in explaining exactly what is meant by a confidence interval or the interpretation of a non-significant result in a test. In branches of engineering concerned with one-off constructions, the application of relative-frequency ideas requires more careful thought and abstraction although, even here, it is often the case that, for example, large numbers of similar components are used.

Students should be encouraged at an early stage to distinguish between a population and a sample, between the (usually unknown) value of a population parameter and its estimate and, of course, between an estimator, that is a random function, and an estimate, that is a particular value. This becomes particularly important from Workbook 40 onwards. Experience shows that students do not always find this easy and often fall into the trap of believing that the data tell us that $\mu = 1.63$ or that the null hypothesis is (definitely) false.

This is, of course, an introductory service course in probability and statistics and it is neither possible nor desirable to attempt to achieve either the breadth or depth that would be given in a course for specialist statistics students. Inevitably there are topics which we do not cover but which may be encountered by engineering students at some time. We feel that the important thing to be achieved in a course such as this is a grasp of the basic ideas and concepts. Once these are understood, coping with new distributions and procedures, perhaps with the aid of suitable computer software and, where appropriate, specialist advice, can often be relatively straightforward.

In the modern world, statistical calculations are usually done with a computer and we expect that students will learn how to do some calculations this way. However it is also useful from a learning point of view to do some, relatively simple, calculations without the aid of a computer program. When a computer is used, it is best to use special statistical software. There are many packages available and many of these are easy to use for beginners, for example Minitab. Some excellent software, e.g. R, is available free of charge on the Web though non-specialist students might not find this quite as convenient to use. Having said this, many students will have access to a spreadsheet program and may prefer to use this. Most of the calculations described in the HELM Workbooks are possible without too much difficulty using popular spreadsheet software. Some engineering students may well be using, other software such as Matlab in other parts of their courses. Matlab also provides

statistical and graphics facilities and, if the students are using it anyway, it may be a sensible option. Few engineering students should have difficulty with the *mathematics* in the statistics Workbooks (especially if they have the benefit of the HELM mathematics Workbooks!). Experience suggests that difficulty is more common with the underlying statistical and probabilistic concepts.

Workbook 35: Sets and Probability

This Workbook serves to underpin all of the probability and statistics covered in the HELM series of Workbooks 35-46. For this reason it is essential that the students fully understand and become conversant with the notation used and the concepts explored. The notation used is standard set notation as found in a myriad of standard textbooks on the subject. Particular attention should be paid to Section 4 - Total Probability and Bayes' Theorem. It is the authors' experience that effort put in to learning the concepts described and fully studying the worked Examples and then completing all of the Tasks and Exercises will pay handsome dividends when later Workbooks concerned with probability and statistics are studied.

Some of the Figures and Examples are in terms of electrical items such as switches and relays wired together in series and/or parallel. Even though the diagrams are straightforward, care should be exercised to ensure that the student is familiar with and understands these diagrams. No knowledge of circuit theory as such is assumed but it is possible that some students may not understand the ideas involved in series and parallel circuit wiring, for example. It is hoped that this will not be a problem for engineering students!

Workbook 36: Descriptive Statistics

The first part of this Workbook, entitled "Describing Data", looks at the basics of calculating means and standard deviations from tabulated data. Many students feel initially that in the age of electronic calculators and computers this is unnecessary. However, it should be born in mind that a familiarity with these basic formulae will pay dividends when later studying Workbook 37: Discrete Probability Distributions and Workbook 38: Continuous Probability Distributions. The first section of Workbook 36 lays the necessary ground for a proper understanding of the formulae and methods used to calculate means and variances for discrete and continuous distributions.

The second section of Workbook 36 is aptly titled "Exploring Data". This is important. The material enables the student to avoid the twin pitfalls of simply taking data either at their face value or accepting them simply as a mass of meaningless figures. By using the techniques outlined in this section of Workbook 36, the student should begin to develop the ability to "look into" a data set and draw tentative conclusions prior to any attempt to analyse the data. This also has the benefit of allowing the student to comment on the possible validity of any conclusions reached using statistical analysis.

Workbook 37: Discrete Probability Distributions

This Workbook looks at the basics of discrete distributions in general and then turns its attention to three distributions in very common use. The binomial distribution is followed by work concerning

the Poisson distribution. This latter distribution is looked at in two distinct ways, firstly as a viable approximation to the binomial distribution, a role in which it can save the user a large amount of tedious arithmetic normally without significantly reducing the accuracy of the answers obtained. The second way in which the Poisson distribution is considered is as a distribution in its own right used to describe the occurrence of events by the so-called Poisson process.

The Workbook concludes with a short section concerning the hypergeometric distribution. This has applications in, for example, acceptance sampling where, because we are sampling without replacement from a finite batch, the results of individual trials, e.g. of whether an item is defective, are not independent. This is in contrast to the usual assumptions underlying the binomial distribution. The distribution of the numbers of defective and acceptable items in a sample from a batch may seem to be of little direct interest. However, when we turn the question round from "Given certain numbers within the batch, what is the probability of observing x?" to the inferential question, "Given observation x, what can we learn about the rest of the batch?" then the hypergeometric distribution turns out to be very relevant. It will also reappear in Workbook 42 in the context of the test for no association in contingency tables.

The notations E for expectation and V for variance are introduced.

Workbook 38: Continuous Probability Distributions

This Workbook follows the same general format as Workbook 37 "Discrete Probability Distributions". Firstly, a look at the basics of continuous distributions in general confirms that the same general pattern may be discerned in the development and use of, for example, the formulae used to calculate means and variances.

The Workbook then turns its attention to two distributions in very common use, the uniform distribution and the exponential distribution. In introducing the exponential distribution, we also begin the development of applications to reliability and lifetimes. The statistical analysis of the nature of the expected lifetime of a product has huge relevance to the industrial applications of statistics and for this reason the exponential distribution is worthy of close study. The concept of a distribution function can be explained in terms of a lifetime distribution and this can then lead to the idea of a probability density function. Analogy with physical mass density can also be helpful.

Workbook 39: The Normal Distribution

The normal distribution is extremely important to all students of statistics, and since a very large number of practical experiments result in measurements, which are closely modelled by this distribution, it is certainly essential that engineers fully understand and can apply this distribution to practical situations. A great many problems can be formulated and solved by using the so-called standard normal distribution. This Workbook will guide the student through a number of applications sufficient to lay a firm basis for future competence in problem solving using the normal distribution. The usual notation of $N(\mu, \sigma^2)$ for a normal distribution with mean μ and variance σ^2 is used.

The second section of the Workbook builds on the foundations laid in Workbook 37 concerning the binomial distribution. Here we look at how the normal distribution can by used to approximate

the binomial distribution provided that the conditions are appropriate. This application can save an enormous amount of arithmetic without loss of accuracy when applied properly.

Finally, this Workbook looks at practical situations whose description involves sums and differences of random variables. Without a working knowledge of the behaviour of these quantities, unacceptable restrictions have to be placed on the range of applications, which an engineer (or anyone else!) is able to consider.

Workbook 40: Sampling Distributions and Estimation

Having looked at probability distributions in Workbooks 37-39, we now move on to inferential statistics. The approach taken to inference is the traditional frequentist approach. Experience shows that attempting to teach *both* frequentist inference *and* the alternative Bayesian inference in one introductory statistic course can lead to confusion and, whether we like it or not, frequentist inference still predominates in much of industry and graduates will be expected to know about significance tests and confidence intervals.

Concepts of inference can often be difficult for students to grasp at first since students are often unused to dealing with uncertainty and questions where the answer is a statement about something which we do not know for certain. For this reason it is important to be careful with the distinctions between, for example, population parameters and sample statistics.

As the title of this Workbook implies, it is concerned with two crucially important aspects of statistics, namely samples, their properties and their use in estimating population parameters. The student should realise that it is often impractical or impossible to deal with a complete population. It is usually the case that the parameters of a population - the mean and variance for example - have to be estimated by using information which is available from a sample or samples taken from the population. Properties of estimators: bias, consistency and efficiency, are introduced. It is important at this stage to make sure that students understand the distinction between population and sample and between estimator and estimate and the concept of a sampling distribution.

It is reasonable to enquire as to the degree of accuracy likely to be involved when a population parameter is estimated. This question is addressed in the Workbook by the construction of confidence intervals for the mean of a population and the variance of a population. The idea of a confidence interval is subtle and not always well understood. Again the concept of properties in repeated sampling is important.

Constructing a confidence interval for the variance of a normal population involves the use of the Chi-squared distribution. An introduction in sufficient detail is given to enable the student to gain a good level of understanding of this distribution and its application to finding confidence intervals involving estimates of the variance of a normal population. Given that a great many practical situations involve statistical estimation it will be appreciated that this Workbook deserves the very close attention of the student.

Workbook 41: Hypothesis Testing

A solid understanding of the theory and techniques contained in this Workbook is essential to any student wishing to apply statistics to real-world engineering problems. Many students find the material challenging, one of the reasons being that work on hypothesis testing is often attempted on the basis of half-understood and half-remembered theory and techniques which necessarily underpin the subject. The student should already be familiar with the material in the HELM statistics series concerning the following topics:

- sampling;

- the normal distribution;

- the binomial distribution;

- the chi-squared distribution.

In this Workbook it is assumed that students have some understanding and a working familiarity with all the above topics. Remember that no attempt is made in this Workbook to teach those topics and that coverage of them will be found elsewhere in the HELM statistics series.

Student's t-distribution and the F-distribution are introduced in this Workbook.

Since many students find some difficulty with this topic, it is strongly suggested that the worked examples are given particular attention and that *all* of the student Tasks are fully worked. The students should not allow themselves to fall into the trap of simply looking up the solution to a problem if they cannot successfully complete it. Suggest that the students keep a record of their attempts to solve problems in order that any lack of understanding may be identified and rectified. Even published scientific literature often contains suggestions of misinterpretation of the results of significance tests. It should be emphasised that a significance test measures (in a particular sense) the evidence against a null hypothesis. A significant result means that we have "strong" evidence against a null hypothesis but not usually certainty. A non-significant result simply means that we do not have such strong evidence. This may be because the null hypothesis is true or it may be that we have just not obtained enough data.

Workbook 42: Goodness of Fit and Contingency Tables

In order that the student understands the first section of this Workbook ("Goodness of Fit") it is essential that the principles of hypothesis testing contained in Workbook 41 are properly understood. In particular, the student should be recommended to revise, if necessary, any work done previously concerning the Chi-squared distribution.

Essentially, the first section of the Workbook is concerned with making decisions as to whether or not a given set of data follow a given distribution (say normal or Poisson) sufficiently closely to be regarded as a sample taken from such a population. If an underlying distribution can be identified, then clearly certain assumptions follow. For example, if a data set is normally distributed then we know that in general we may calculate parameters such as the mean and variance by using certain standard formulae. If a data set follows the Poisson distribution then we may assume that the population mean and variance are numerically identical.

The second section of the Workbook ("Contingency Tables") is concerned with situations in which members of samples drawn from a population can be classified by more than one method, for example the failure of electronic components in a system installed in a machine and the positions in a machine in which they are mounted. The Workbook discusses how such information may be presented and follows this by a discussion involving hypothesis tests to decide (using the example outlined above) whether or not there is sufficient evidence to conclude that failure of a component is related to position in the machine.

Workbook 43: Correlation and Regression

Section 43.1 introduces the study of regression analysis, that is, the study of possible predictive or explanatory relationships between variables. The student should note that the real work of this Section deals with linear regression on one variable only. In the real world, regression may be multiple (i.e. with several explanatory variables) and may be non-linear. A very short introduction to non-linear regression is given at the end of 43.1 and is introduced for the sake of completeness only - simply to signal to the student that non-linear regression does exist and that the real world is often anything but linear!

Modern computer software makes the calculations for multiple regressions and many nonlinear regressions readily available but an understanding of the basic ideas, gained through looking at simple cases, is essential to interpret the results.

The main technique used to study linear regression is the method of least squares. In order that this topic be fully understood, it is necessary to use some multi-variable calculus. The Workbook deliberately ignores this and simply presents the equations resulting from the use of this technique.

Section 43.2 is concerned with the topic of correlation. Two common methods of measuring the degree of a possible relationship between two random variables are considered, these are Pearson's r and Spearman's R. The methods of calculation given will result in a numerical value being obtained. The strength of evidence for the existence of a relationship may be measured by performing a significance test. The importance of the normality assumption for the usual test of Pearson's r should be emphasised. In cases where this assumption cannot be made or where a relationship may be monotonic but not linear Spearman's R should be used. It is worth noting that significance tests involving both measures of correlation involve the use of Student's t-distribution and that it may be worth revising this topic before a study of correlation is attempted.

Workbook 44: Analysis of Variance

This Workbook covers three topics, one-way analysis of variance, two-way analysis of variance and an introduction to experimental design. In order to obtain the maximum benefit from this Workbook, the students should ensure that they are familiar with the general principles of hypothesis testing. In particular, a working knowledge of the F-distribution is essential. This groundwork is covered in Workbook HELM 41-Hypothesis Testing.

The Workbook starts with one-way ANOVA and the introduction makes clear the advantages accruing from a technique enabling us to *simultaneously* compare several means. On reading the introduction,

it should be clear to the student that not only is the amount of work needed to compare several means drastically reduced but also that use of ANOVA gives the required significance level whereas use of a collection of pairwise tests would distort the significance level. When working through the learning material, students should ensure that they clearly understand the difference between the phrases "variance *between* sample" and "variance *within* samples".

While one-way ANOVA considers the effect of only one factor on the values taken by a variable, two-way ANOVA considers the simultaneous effect of two or more factors on a variable. Two distinct cases are considered, firstly when possible interaction between the factors themselves is ignored and secondly when such interaction is taken into account. The Workbook concludes with a short introduction to experimental design. This section is included to encourage students to consider the design of any engineering based experimental work that they may be required to undertake and to appreciate that the application of statistical methods to engineering begins with the design of an experiment and not with the analysis of the data arising from the experiment.

Workbook 45: Non-parametric Statistics

This Workbook looks at hypothesis testing under conditions, which are such that we do not, or cannot, because of a lack of evidence, make the claim that the distributions we are required to deal with follow a specific distribution. Commonly, we may assume that data follow a normal distribution and apply say, a t-test under appropriate conditions. Such assumptions are not always possible and we have to use distribution-free tests. Such tests are usually referred to as non-parametric tests since they do not refer to distribution parameters, for example the mean of a normal distribution. This Workbook assumes a familiarity with the techniques of hypothesis testing covered in Workbook HELM 41, in particular the t-test. A working knowledge of the binomial distribution (HELM 37) is also essential background reading. The Workbook considers the application of non-parametric testing to situations involving single samples and situations involving two samples.

A common error is to suppose that there are such things as "parametric data" and "non-parametric data". It is not the data, which are "parametric" or "non-parametric", of course, but the procedures, which are applied, and the assumptions underlying them. It is hoped that this Workbook will convey an understanding of the different assumptions which are made, just what it is that is being tested and when it is appropriate to use a nonparametric test. The tests which are specifically addressed, the sign test, the Wilcoxon signed-rank test and the Wilcoxon rank-sum test (which is equivalent to the Mann-Whitney U-test) are straightforward in themselves but the assumptions underlying them and their interpretation are not always well grasped.

Workbook 46: Reliability and Quality Control

This Workbook is split into two distinct sections, the first covering the topic of reliability and the second giving an introduction to the crucially important topic of quality control. Necessary background reading for this Workbook consists of general probability (HELM 35) and continuous probability distributions (HELM 38). In Workbook 46 the study of reliability begins with a look at lifetime distributions and progresses to work concerning system reliability with some surprising results. For example while it may be taken to be intuitively obvious that a set of electronic components wired in

series will be as reliable as the least reliable member, this turns out not to be the case - the set turns out to be less reliable than its least reliable member!

The section concerning quality control starts with a very brief history of the subject and introduces the student to some essential elementary control techniques including simple quality control charts, R-charts and Pareto charts. Elementary trend detection is considered via the use of "standard" checks, which the student may apply to, given data sets.

The material in this Workbook will have a clear relevance for many engineering students. Tutors might like to consider using some of the material earlier in the sequence of Workbooks if this seems appropriate to maintain interest and motivation.

Workbook 47: Mathematics and Physics Miscellany

Section 47.1 "Dimensional Analysis in Engineering" sets out several examples, which show the use of dimensional analysis in the analysis of physical systems. The standard examples such as the simple pendulum are treated in a detailed manner, so as to emphasize that dimensional analysis has several "blind spots"; in particular the unknown "constant" which appears in standard simple treatments must actually be regarded as a **dimensionless quantity** and so could be an arbitrary function of an angle, of a Mach number, etc. In several cases the usual simple approach is compared with the Buckingham approach, which makes the dimensionless quantities the dominant quantities in the description of a physical system. The examples treated gradually increase in complexity, with the theory of fluids providing examples for which the set of relevant physical variables is sufficiently rich to provide two different dimensionless quantities in the Buckingham approach.

[See the references at the end of this Section in the Workbook, which includes suggested Google search keywords.]

Section 47.2 "Mathematical Explorations" provides an introduction to a range of techniques, which provide useful alternatives to those of the other Workbooks. One theme, which appears repeatedly, is that of the use of power series methods in tackling problems, which are often treated, by the use of ordinary algebra. The tutor might note that there is an implicit underlying theme here which is worth emphasizing in discussions: when we find the Maclaurin series for $\tan(x)$ by studying the equation $dy/dx = 1 + y^2$ what we are actually doing is finding a power series solution of a differential equation. For functions, which are already familiar, this aspect is not obtrusive but these "known" examples can be used to motivate an approach to the study of power series solutions of more general differential equations, especially for new cases in which we do not know the solution before we start. For example, the power series approach is a useful tool in finding the energy levels associated with the Schrödinger equation in quantum mechanics, particularly for cases in which the equation cannot be solved by the simple analytic methods set out in simple textbooks.

Section 47.3 "Physics Case Studies" has eleven items. Many engineering problems are based upon fundamental physics and require mathematical modelling for their solution. This Section contains a compendium of case studies involving physics (or related topics) as an additional teaching and learning resource beyond those included in the previous HELM Workbooks.

Each case study will involve several mathematical topics; the relevant HELM Workbooks are stated at the beginning of each case study.

Table 1: Physics topics, related Mathematical topics and Workbooks

	Physics Case Study Title	Mathematical Topics	Related Workbooks
1	Black body radiation 1	Logarithms and Exponentials; Numerical integration	6, 31
2	Black body radiation 2	Logarithms and Exponentials; Numerical solution of equations	6, 31
3	Black body radiation 3	Logarithms and Exponentials; Differentiation	6, 11
4	Black body radiation 4	Logarithms and Exponentials; Integration; Numerical integration	6, 13, 31
5	Amplitude of a monochromatic optical wave passing through a glass plate	Trigonometric functions; Complex numbers; Sum of geometric series	4, 10, 16
6	Intensity of the interference field due to a glass plate	Trigonometric functions; Complex numbers	4, 10
7	Propagation time difference between two light rays	Trigonometric functions	4
8	Fraunhofer diffraction through an infinitely long slit	Trigonometric functions; Complex numbers; Maxima and minima	4, 10, 12
9	Fraunhofer diffraction through an array of parallel infinitely long slits	Trigonometric functions; Exponential functions; Complex numbers; Maxima and minima; Geometric series; Fourier transform	4, 6, 10, 12, 16, 24
10	Interference fringes due to two parallel infinitely long slits	Trigonometric functions; Complex numbers; Maxima and minima; Maclaurin series	4, 10, 12, 16
11	Acceleration in polar coordinates	Vectors; Polar coordinates	9, 17

Time allocation

To work through the whole Workbook will require at least twenty hours of independent study. However it would be more normal (and preferable) to use it to 'dip-in-and-out', and to follow up additional examples of modelling using particular techniques for an hour or two at a time.

Format

The eleven Physics Case Studies in Workbook 47 have a common format (with rare minor variations), This is the same as that used for the Engineering Examples in Workbooks 1 to 34. The section headings are:

Introduction
This consists of a paragraph or two of background information, setting the context and stating essential engineering information, definitions and fundamental concepts.

Problem in words
This consists of a statement of the problem in words including the purpose of the model.

Mathematical statement of problem
Here the problem is expressed in mathematical form including notation, assumptions and strategy.

Mathematical analysis
This part gives the solution to the problem or explains how it can be solved using techniques in the indicated Workbooks.

Interpretation
This interprets the mathematical result in engineering terms and includes at least a statement of the result in words. If appropriate, there are comments about whether results make sense, mathematical points, indications of further extensions or applications and implications.

Workbook 48: Engineering Case Studies

This Workbook offers a compendium of twenty 'Engineering Case Studies' providing additional teaching and learning resources to the sixty-seven 'Engineering Examples' included in Workbooks 1 to 34.

Table 2 below summarises the engineering contexts, the mathematical topics and the relevant HELM mathematics Workbooks for these Case Studies. It should be possible to use this Workbook to reinforce notions of modelling using a wide cross section of mathematical techniques. The more elementary mathematical topics and relevant Workbooks are not usually mentioned - for example basic functions (Workbook 2) pervades every case Study and so is omitted. However, where there is significant algebraic manipulation (Workbook 1) or equation rearrangement (Workbook 3) this has been reflected in the table.

The Case Studies have been grouped together by broad engineering topic and not by mathematical topic or difficulty or length. (The shortest and most straightforward is 11.)

Table 2: Engineering topics, related Mathematical topics and Workbooks

	Engineering Topic	**Mathematical Topics**	**Related Workbooks**
1	Adding sound waves	Trigonometry; Second order ODEs	4, 19
2	Complex representations of sound waves and sound reflection	Complex numbers; Trigonometry	4, 10
3	Sensitivity of microphones	Definite integrals (function of a function)	13, 14
4	Refraction	Algebra; Equations; Trigonometry; Differentiation	1, 3, 4, 11
5	Beam deformation	Algebra; Equations; Inequalities; Trigonometry; Definite integrals; First order ODEs	1, 3, 4, 13, 19
6	Deflection of a beam	Equations; Definite integrals; ODEs; Laplace transforms	3, 13, 19, 20
7	Buckling of columns	Trigonometric identities; Determinants; Matrices; Integration; Fourth order ODEs	4, 7, 8, 13, 19
8	Maximum bending moment for a multiple structure	Algebra; Equations; Inequalities; Maxima and minima (differentiation)	1, 3, 12
9	Equation of the curve of a cable fixed at two endpoints	Trigonometric identities; Hyperbolic functions; Differentiation; Integration; First order ODEs	4, 6, 11, 13, 19
10	Critical water height in an open channel	Algebra; Equations; Derivative of polynomials; Maxima and minima (differentiation); Integration	1, 3, 11, 12, 13
11	Simple pendulum	Algebra - rearranging formulae	1
12	Motion of a pendulum	Trigonometric functions; Differentiation; Maclaurin's expansion; Contour plotting for a function of two variables; Second order ODEs	4, 11, 16, 18, 19
13	The falling snowflake	Integration; First order ODEs; Second order ODEs; Vector differentiation	13, 19, 28
14	Satellite motion	Trigonometry; Equations of conics in polar coordinates; Second order ODEs; Vector differential calculus; Polar coordinate system	4, 17, 19, 28
15	Satellite orbits	Trigonometry; Equations of conics in polar coordinates; Second order ODEs; Vector differential calculus; Polar coordinate system	4, 17, 19, 28

16	Underground railway signals location	Trigonometry; Differentiation of inverse trigonometrical functions; Geometry (arc length); Partial differentiation	4, 11, 17, 18
17	Heat conduction through a wine cellar roof	Trigonometry; First order ODEs; Second order ODEs; Fourier series; Partial differential equations	4, 19, 23, 25
18	Two-dimensional flow past a cylindrical obstacle	Algebra - rearranging formulae; Trigonometry; Polar coordinates	1, 4, 17
19	Two-dimensional flow of a viscous liquid on an inclined plate	Trigonometry; Integration; Polar coordinates; Partial differentiation; Partial differential equations; Vector differential calculus; Dimensional analysis	4, 13, 17, 18, 19, 28, 47
20	Force on a cylinder due to a two-dimensional streaming and swirling flow	Algebra; Equations; Trigonometry; Logarithmic functions; Integration; Orthogonality relations of trigonometric functions; Partial differentiation; Polar coordinates; Surface integrals	1, 3, 4, 6, 13, 17, 18, 28, 29

Time allocation

To work through the whole Workbook will require at least fifty hours of independent study. However it would be more normal (and preferable) to use it to 'dip-in-and-out', and to follow up additional examples of modelling using particular techniques for an hour or two at a time.

Format

The twenty Engineering Case Studies in Workbook 48 have a common format (with rare minor variations). This is the same as that used for the Engineering Examples in Workbooks 1 to 34. The section headings are:

Introduction
This consists of a paragraph or two of background information, setting the context and stating essential engineering information, definitions and fundamental concepts.

Problem in words
This consists of a statement of the problem in words including the purpose of the model.

Mathematical statement of problem
Here the problem is expressed in mathematical form including notation, assumptions and strategy. Mathematical analysis This part gives the solution to the problem or explains how it can be solved using techniques in the indicated Workbooks.

Interpretation
This interprets the mathematical result in engineering terms and includes at least a statement of the result in words. If appropriate, there are comments about whether results make sense, mathematical points, indications of further extensions or applications and implications.

Where possible there are comments on the sensitivity of the analysis to input data and the necessary numerical accuracy of the outputs.

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